BUCKLING OF BAR BY WAVELET –GALERKIN METHOD

A

THISIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF TECHNOLOGY

IN

CIVIL ENGINEERING

WITH SPECIALIZATION IN

STRUCTURAL ENGINEERING

BY

SUJI P

ROLL NO: 210CE2027

NATIONAL INSTITUTE OF TECHNOLOGY
ROURKELA – 769008.
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CERTIFICATE

This is to certified that the thesis entitled, **"BUCKLING OF BAR BY WAVELET GALERKIN METHOD"** submitted by **SUJI P** bearing Roll No**-210CE2027** in partial fulfillment of the requirement for the award of the degree of **Master of Technology** in **Civil Engineering** with the specialization of **"Structural Engineering"** of National Institute of Technology, Rourkela is a record of bonafied work carried by her under my supervision and guidance in academic year 2011-2012.

This thesis fulfils the requirements relating to the nature and standard work for the award of Master of Technology in Civil Engineering. To the best of my knowledge the matter embodied in the thesis has not been submitted to any other Universit/Institute for the award of any degree or diploma.

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ABSTRACT

Wavelet are used in data compression, signal analysis and image processing and also for analyzing non stationary time series. Wavelets are the functions which satisfy certain mathematics requirements and are used in other functions. The use of wavelets in mechanics can be viewed from two perspectives, first the analysis of mechanical response for extraction of modal parameters, damage measures, de-noising etc and second the solution of the differential equations governing the mechanical system. Wavelet theory provides various basis functions and multi-resolution methods for finite element method. Wavelet-based beam element can be constructed by using Daubechies scaling functions as an interpolating function. In the present thesis, the compactly supported Daubechies wavelet based numerical solution of boundary value problem has been presented for the instability analysis of prismatic members. This problem can be discretized by the Wavelet-Galerkin method. The evaluation of connection coefficients plays an important role in applying wavelet galerkin method to solve partial differential equations. The buckling problem of axially compressed bars by using Wavelet-Galerkin method is explained in this thesis. The comparisons are made with analytical solutions and with finite element results. The present investigation indicated that wavelet technique provides a powerful alternative to the finite element method.

The thesis has been presented in six number of chapters. **Chapter 1** deals with the general introduction to wavelets and different types of wavelet families and their properties. The review of literature confining to the scope of study has been presented in **chapter 2**. The **Chapter** 3 deals with the properties of Daubechies wavelets and determination of scaling function, wavelet function, filter coefficients and moments of scaling function for different order of Daubechies wavelets. The computation of connection coefficients are described in **Chapter 4. Chapter 5** deals with the Wavelet Galerkin method and the buckling problem of prismatic bar by using wavelet Galerkin method. It also includes the numerical results obtained from the present work, and comparisons made with analytical solutions and with finite element results. The **Chapter6** concludes the present investigation. An account of possible scope of extended study has been presented to the concluding remarks. At last, some important publications and books referred during the present investigation have been listed in **Reference** section.

KEYWORDS: Introduction to wavelet theory, Daubechies wavelets scaling function, connection coefficients, wavelet galerkin method, buckling of bars.

CONTENTS

Title	Page No.
ACKNOWLEDGEMENTS	i
ABSTRACT	ii
TABLE OF CONTENTS	iii
LIST OF TABLES	vii
LIST OF FIGURES	viii
LIST OF SYMBOLS	x

CHAPTER 1 INTRODUCTION

1.1.	Overview	1
1.2.	Introduction to wavelets	1
1.3.	Wavelet Transform	2
1.3.1.	Discrete Wavelet Transform	2
1.3.2.	Continuous Wavelet Transform.	3
1.3.3.	Wavelet Transform Versus Fourier Transform.	4
1.3.4.	Similarities between Fourier and Wavelet transform	4
1.3.5.	Dissimilarities between Fourier and Wavelet transform.	4
1.4.	Types of Wavelets	5
1.4.1.	Haar Wavelets.	6
1.4.2.	Daubechies Wavelets.	7
1.4.3.	Coiflets wavelets.	10
1.4.4.	Symlets Wavelets.	11

1.4.5. Morlets Wavelets. 11

1.4.6. Mexican Hat Wavelets. 12

1.4.7. Biorthogonal Wavelets..13

1.5. Introduction to Wavelet Galerkin Method 14

CHAPTER 2 REVIEW OF LITERATURE

2.1. Introduction 15

2.2. Review of Literature on Wavelet Methods 15

2.3. Review of Literature on Wavelet Galerkin Method 18

2.3.1. Review of Literature on connection coefficient.....................................21

CHAPTER 3 PROPERTIES OF WAVELETS

3.1. Introduction..22

3.2. Daubechies Compactly supported Wavelets...22

3.3. Wavelet Function 22

3.4. Scaling Function 25

3.4.1. Filter Coefficients 27

3.4.2. Derivatives of Scaling function. 29

3.4.3. Integrals of Scaling Function. 31

3.4.4. Moments of scaling Function...34

3.4.5. Interpolation of Scaling Function. 34

CHAPTER 4 CONNECTION COEFFICIENTS

4.1. Introduction....36

4.2. Connection coefficients 36

4.3. Determination of Connection coefficients....37

4.3.1. Two- Term connection Coefficients....37

4.3.2. Three – Term connection coefficients. 42

CHAPTER 5 NUMERICAL EXAMPLE

5.1. Introduction 45

5.2. Critical Load 45

5.3. Wavelet Galerkin method. 46

5.4. Exact Method....47

5.5. Numerical example. 50

5.5.1. Clamped supported condition. 51

5.5.2. Simply supported condition....54

5.6. Result and Discussion....56

CHAPTER 6 CONCLUSION

6.1. Conclusion. 59

6.2. Scope of Future work....60

REFERENCES. 61

LIST OF TABLES

Table 1.1: Values for Nr and Nd for Biorthogonal wavelets......................................14

Table 3.1: Daubechies wavelet coefficients..23

Table 3.2: Daubechies wavelet filter coefficients...27

Table 3.3: Scaling function...29

Table 3.4: Derivatives and Integrals of scaling function...34

Table 4.1: Values of Connection coefficients...42

Table 5.1.Values of Buckling load 56

Table 5.2: Daubechies filter coefficient for D6. 57

LIST OF FIGURES

Title **Page No**

Figure1.1: Wavelets 2

Figure1.2:Mallat forward discrete transform 3

Figure1.3:Windowed Fourier transform 5

Figure 1.4:Haar wavelet scaling function 6

Figure 1.5: Haar wavelet function 7

Figure 1.6:Daubechies wavelets for 6 iteration 8

Figure 1.7:Different types Daubechies wavelet family 9

Figure 1.8:Coiflets wavelets with order 2 10

Figure 1.9Coiflets wavelets with order 5 11

Figure 1.10:Morlet wavelet function 12

Figure 1.11:Mexican hat wavelet function 13

Figure 3.1: Wavelet function for D4,D6,D8,D10,D12,D22 24

Figure 3.2:Scaling function for D4,D6,D8,D10,D12,D22 28

Figure 3.3:First derivative of scaling function for D6 31

Figure 3.4: Second derivative of scaling function 31

Figure 3.5: Interpolate IN6 scaling function 35

Figure 3.6: Interpolate IN8 scaling function 35

Figure 5.1: Effective length for different support..50

Figure 5.2: percentage of error for different level of resolution.................................57

Figure 5.3: Scaling function (db3)..58

Figure 5.4: Derivative of scaling function 58

LIST OF SYMBOLS

The principle symbols in this thesis are presented for easy reference. A single symbol is used different meaning depending on the contest and defined in the text as they occur.

English

a_k	Filter coefficients
CWT	Continuous wavelet transform
DWT	Discrete wavelet transform
db	Daubechies wavelet
M	Vanishing moments
N	Order of Wavelets
J	Level of resolution
P	Compressive load
E	Modulus of elasticity
I	Moment of inertia
m_k	Moments of scaling function
W	Middle point deflection
L	Length of bar
$\boldsymbol{\Delta}$	Two-term connection coefficients
Ω	Three –term connection coefficients
δ	Kronecker delta function
Ψ	Wavelet function
φ	Scaling function
γ	Integral of scaling function

CHAPTER 1

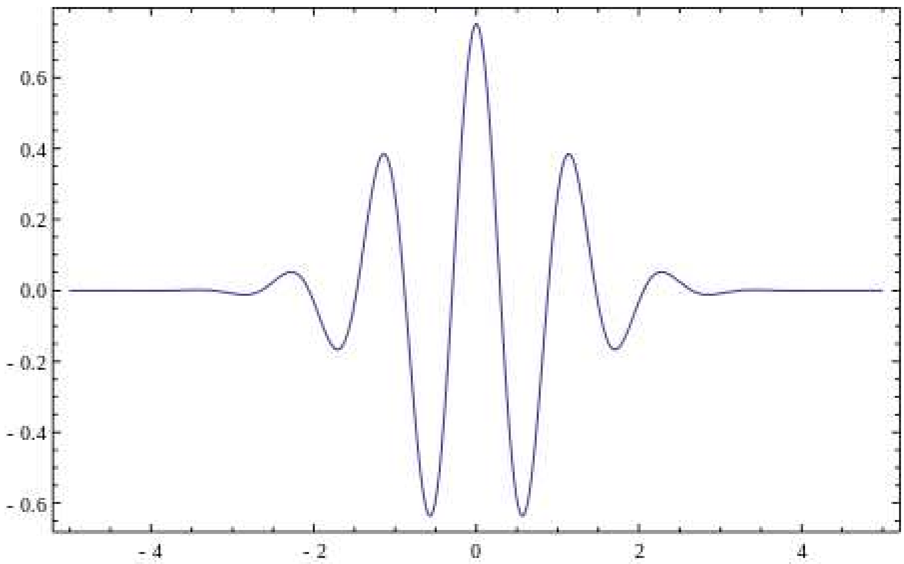
INTRODUCTION

1.1. OVERVIEW

In recent years, the buckling analysis of structures is carried out by using finite difference and finite element method. Here, we explain buckling analysis of structures by the wavelet - Galerkin method. The Wavelet Galerkin method is used for solving partial differential equations and differential equations. The connection coefficients play an important role in applying Wavelet-Galerkin method. Daubechies wavelets have been successfully used in as a base function in wavelet galerkin method, due to their compact support, orthogonality and multiresolution properties.

1.2. INTRODUCTION TO WAVELETS

The word wavelet is derived from a French word *ondelette* means ‘small wave’. Wavelet analysis was developed in the mathematical literature in 1980. Wavelet can be used in data compression, signal analysis and image processing. Wavelets are also used for analyzing non-stationary time series. Wavelets are those functions which satisfy certain mathematical requirements and are used in other functions. The use of wavelets in mechanics can be viewed from two perspectives; first, the analysis of mechanical response for extraction of modal parameters, damage measures, de-noising etc, and second, the solution of the differential equations governing the mechanical system. The fundamental idea about wavelets is to analyze according to scale. The wavelet based numerical solution has recently developed the theory and application of partial differential equations. Wavelet analysis is a numerical concept which allows representing a function in terms of basic functions, called wavelets, which are localized both in location and scale. Among the wavelet approximation, the Wavelet -Galerkin technique is most frequently used scheme now a day. Wavelet theory provides various basis functions and multi-resolution methods. In finite element method, wavelet-based beam element can be constructed by using Daubechies scaling functions as an interpolating function. Since the nodal lateral displacements and rotations are used as element degrees of freedom, the connection between neighboring elements and boundary conditions can be processed simply as done for traditional elements.



FIGURE(1.1): Wavelets

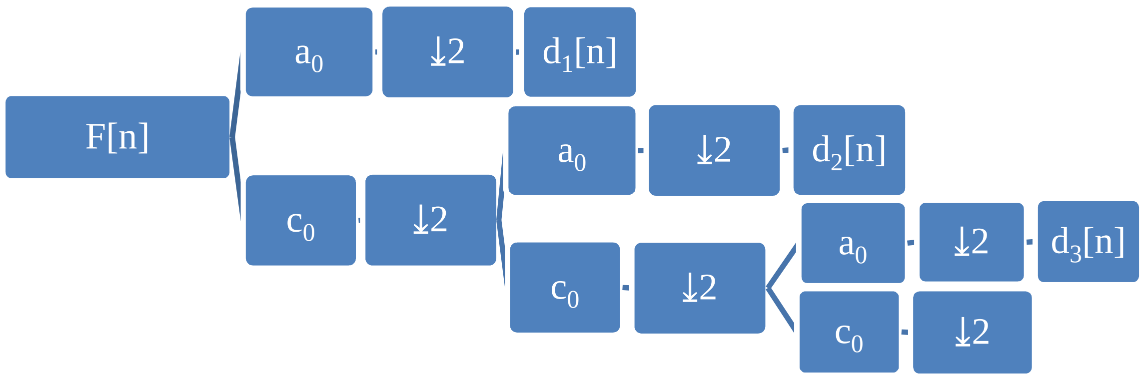
1.3. WAVELET TRANSFORM

According to Daubechies, wavelet transform can be defined as, a tool that cuts up data, functions or operators in to different frequency components, and then studied each resolution component with a resolution matched to its scale. Wavelet transform are transform which provides a time frequency representation of the signal. The wavelet transform is developed to overcome the short- coming of the short Fourier transform. Wavelet transform is also used for multiresolution technique.

1) Discrete wavelet transforms (DWT)
2) Continuous wavelet transforms (CWT)

1.3.1. Discrete Wavelet Transforms (DWT)

DWT is a Multiresolution representation of a finite length discretized signal from fine to coarse scale. It is impossible to analyze a signal using all wavelet coefficients, so one may wonder if it was sufficient to extract a discrete set of the upper half plane to be enough to reconstruct a signal from the wavelet coefficients. Discrete wavelet transform has been performed using a fast algorithm referred to a as Mallat transform[25]. Figure (1.2) shows Mallat forward transform. The discrete wavelets transform which convert discrete signals to discrete wavelet representation.



Figure(1.2): Mallat Forward Discrete Wavelet Transform

1.3.2. Continuous Wavelet Transform (CWT)

The Continuous Wavelet Transform (CWT) is as given in equation (1.1),

$$X_{w,t} = \frac{1}{\sqrt{|s|}} \int x(t)\, \Psi\left(\frac{t-\tau}{s}\right) dt \qquad (1.1)$$

where,

x(t) = signal

$\Psi(t)$ = mother wavelet or wavelet function

τ = translation parameter

s = scale parameter

CWT is obtained by using computers sampling the time-scale plane. All the wavelet function used in the transformation are derived from mother wavelet by translation and dilation.

1.3.3. Wavelet Transforms versus Fourier Transforms

1.3.3.1.Similarities between Fourier and Wavelet transform

The fast Fourier transform (FFT) and the discrete wavelet transform (DWT) are both linear operations, they generated a data structure that contains '*log2 n*' segments of several lengths, usually transforming it into a different data vector of length *2n*. The inverse transform for both the FFT and the DWT matrices are the transpose of the original. As a result, both transforms can be represented as a rotation function space to a different domain. For the FFT, contains basis functions that are sines and cosines. For the wavelet transform, contains more complicated basis functions called wavelets, or analyzing wavelets, or mother wavelets.

1.3.3.2.Dissimilarities between Fourier and Wavelet Transforms

The most interesting dissimilarity between FFT and wavelet transforms are individual wavelet functions are localized in space. In wavelet transform have number of useful applications such as data compression, and removing noise from time series. The differences between the Fourier transform and the wavelet transform in the time-frequency resolution is to look at the basis function of the time-frequency plane . Figure (1.3) presented a windowed Fourier transform, where the window was simply a square wave. The square wave window truncated the sine or cosine function to a window of a particular length,because a single window is used for all frequencies in the wavelet Fourier transform, the resolution of the analysis was the same at all locations in the time-frequency plane.

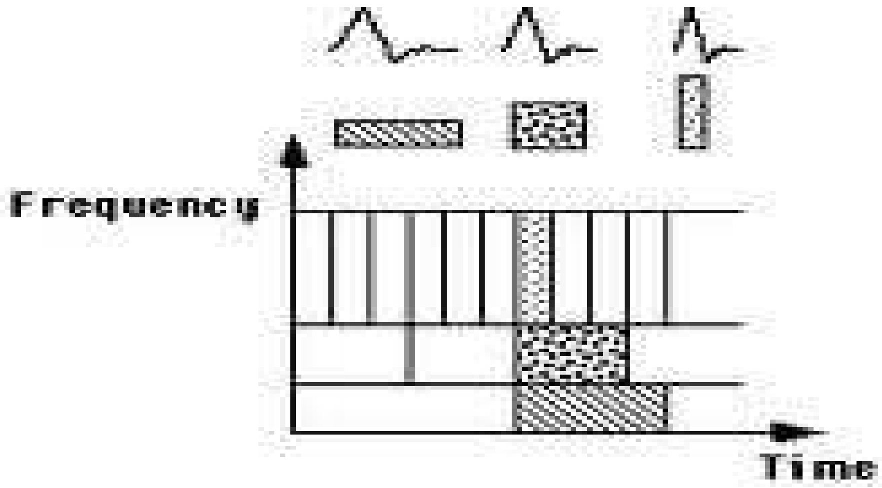


Figure (1.3): Windowed Fourier Transform

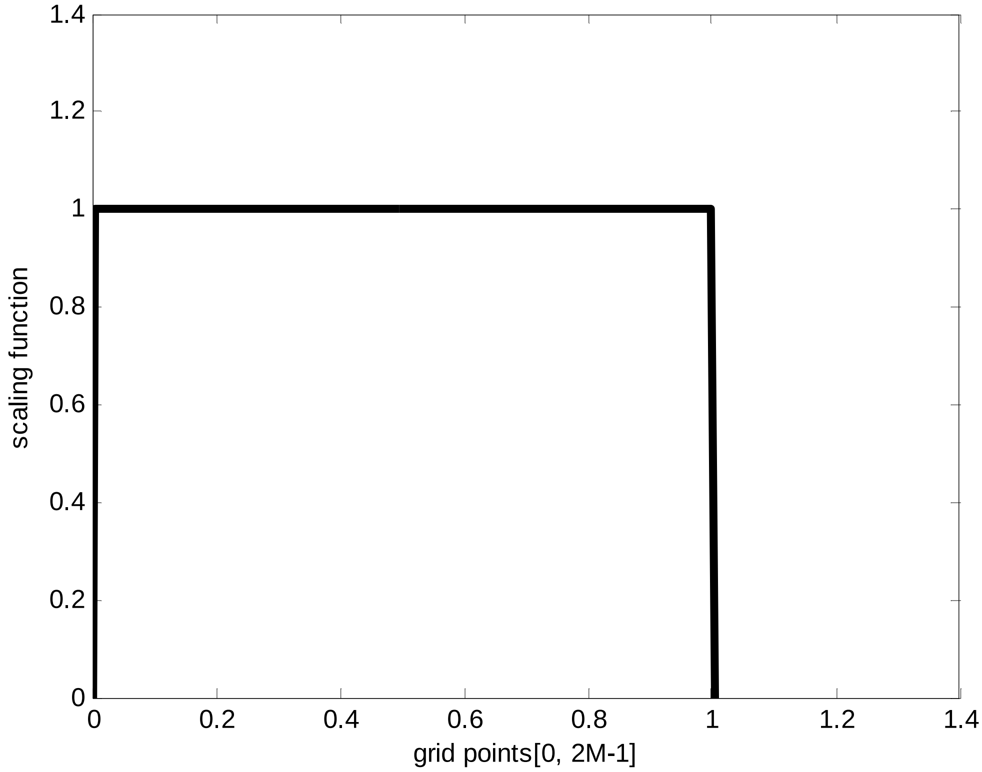
1.4. TYPES OF WAVELETS

The followings are the different types of wavelets.

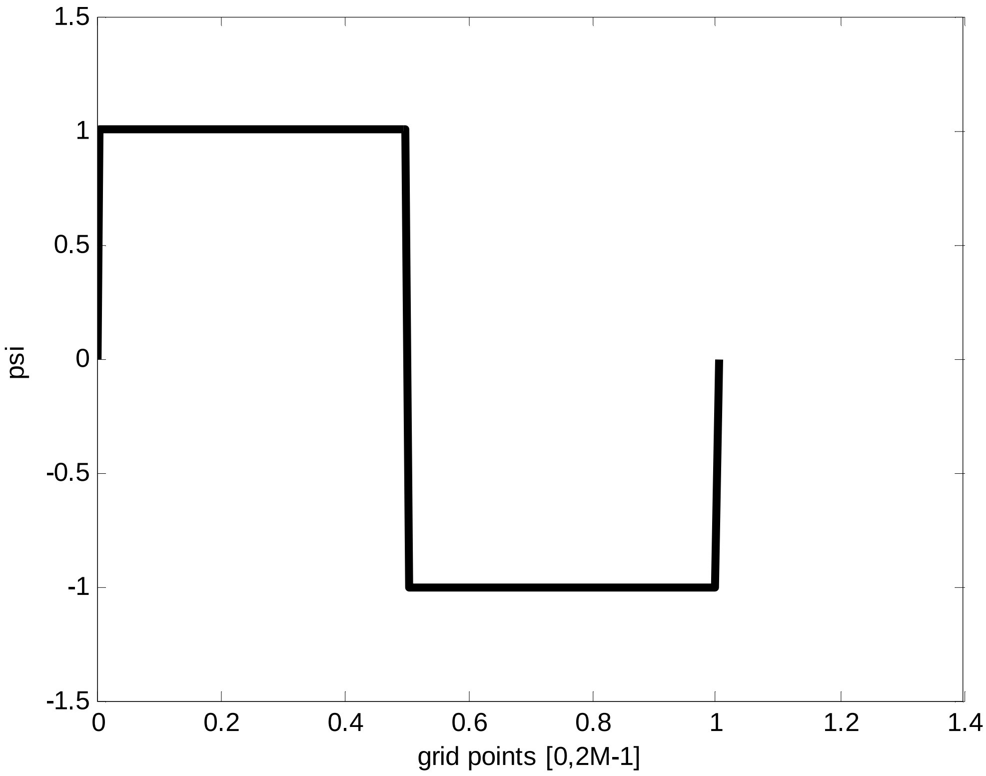
- Haar wavelet
- Daubechies wavelet
- Coiflets wavelet
- Symlets wavelets
- Biorthogonal wavelets
- Morlet wavelets
- Mexian hat wavelet

1.4.1. Haar Wavelet

The first order Daubechies wavelets are known as Haar wavelet. Haar wavelet is the simplest and oldest wavelets for all wavelets. It has been used in various mathematical fields. The Brownian motion can be defined by Haar wavelet. Haar wavelet is the dilation and translation of wavelet function. Any discussion of wavelet begins with Haar wavelet, they are the first and simplest wavelets. Haar wavelet is discontinuous and used as a step function. It represents the same wavelet Daubechies db1. Figure (1.4) and figure (1.5) represence the scaling function and wavelet function for Haar wavelets respectively.



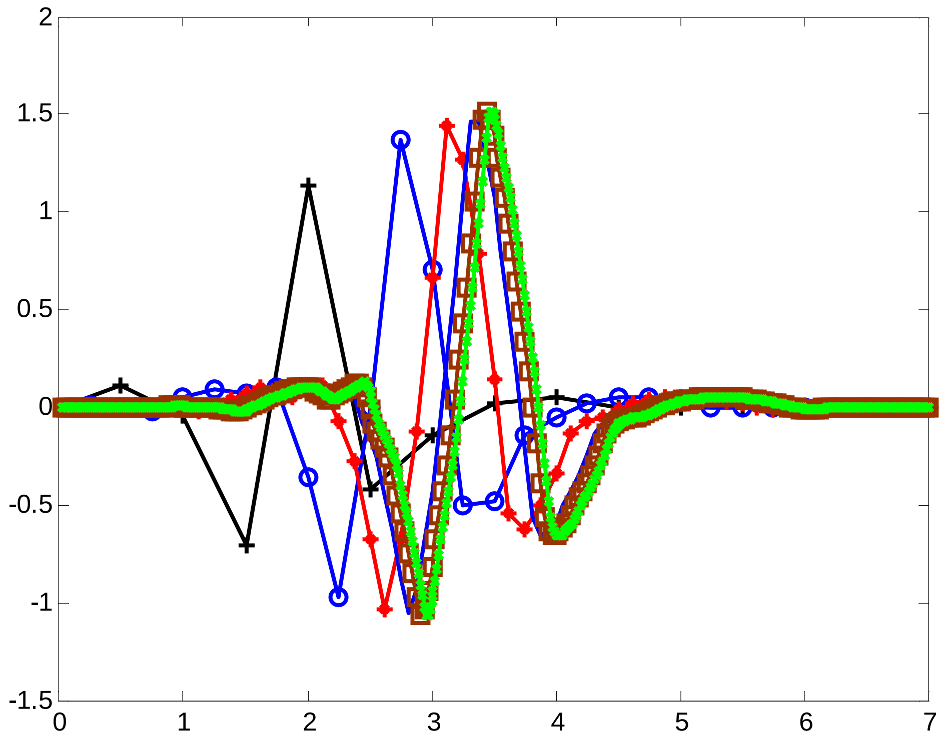
Figure(1.4): Haar Wavelet Scaling Function



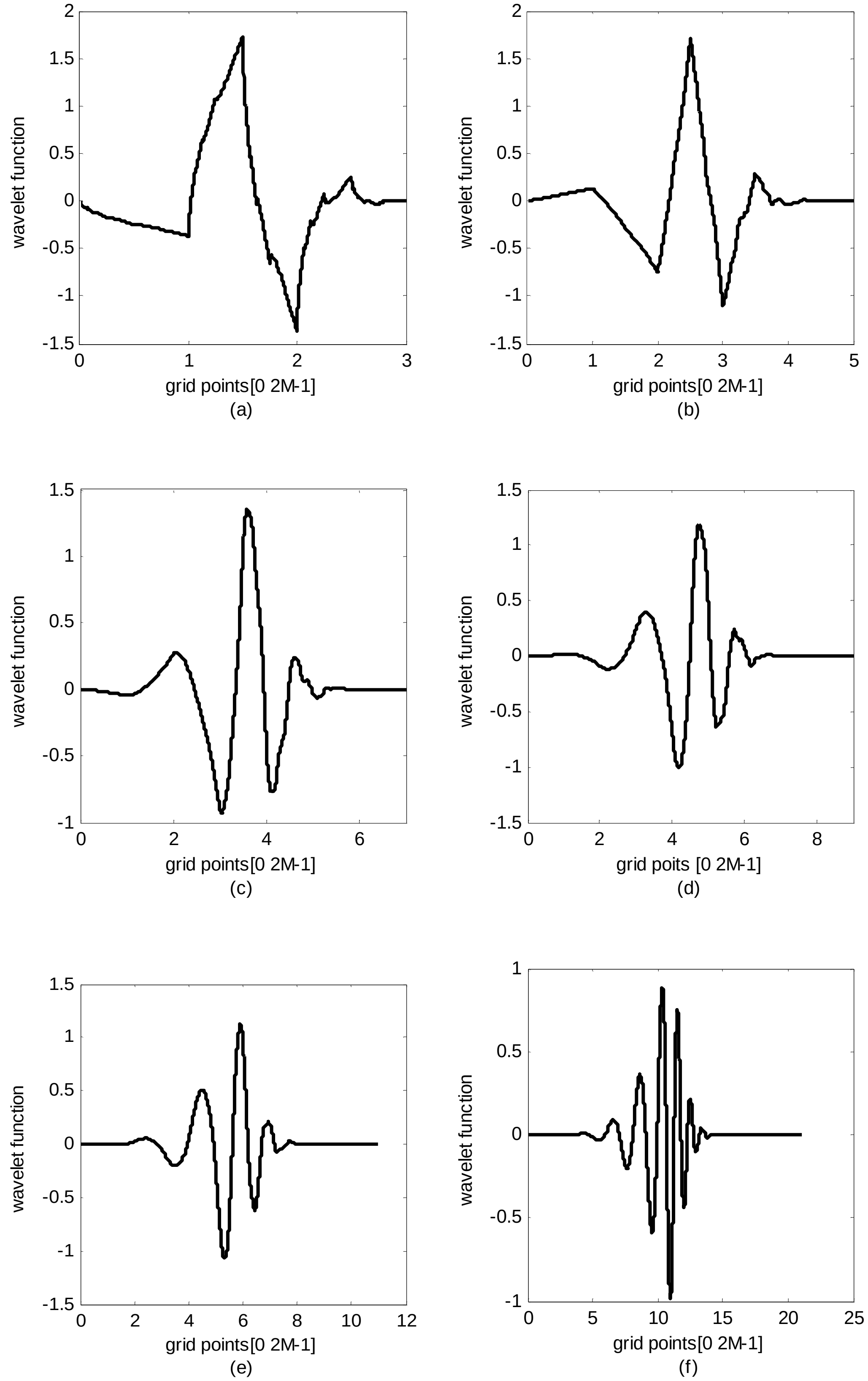
Figure(1.5): Haar Wavelet Function

1.4.2. Daubechies Wavelet

The family of this type of wavelet constructed by Daubechies includes members from highly localized to highly smooth. The orthonormal basis of compactly supported wavelet is generated by dilation and translation of wavelet function. The regularity order increases linearly with the support condition. All the examples of compactly supported wavelet bases are conspicuously non symmetric in contrast to infinitely support wavelet bases. Compactly supported wavelets have been successfully applied in numerical simulation. Daubechies proposed orthogonal compactly supported wavelets are known as Daubechies wavelet. The first order Daubechies wavelet becomes the well known Haar wavelet. Daubechies orthogonal wavelets db1 to db20 are commonly used. Figure (1.7) shows different types of Daubechies wavelet family. The higher order Daubechies function are not easy to describe with an analytical expression. The order of Daubechies functions are denoted by the vanishing moments. The larger the number of vanishing moments, better the frequency localization of the decomposition.. Daubechies wavelets are commonly used in solving a broad range of problems the order of the wavelet function can be compared to the order of a linear filter. Figure (1.6) shows Daubechies wavelet function for 6 iteration.



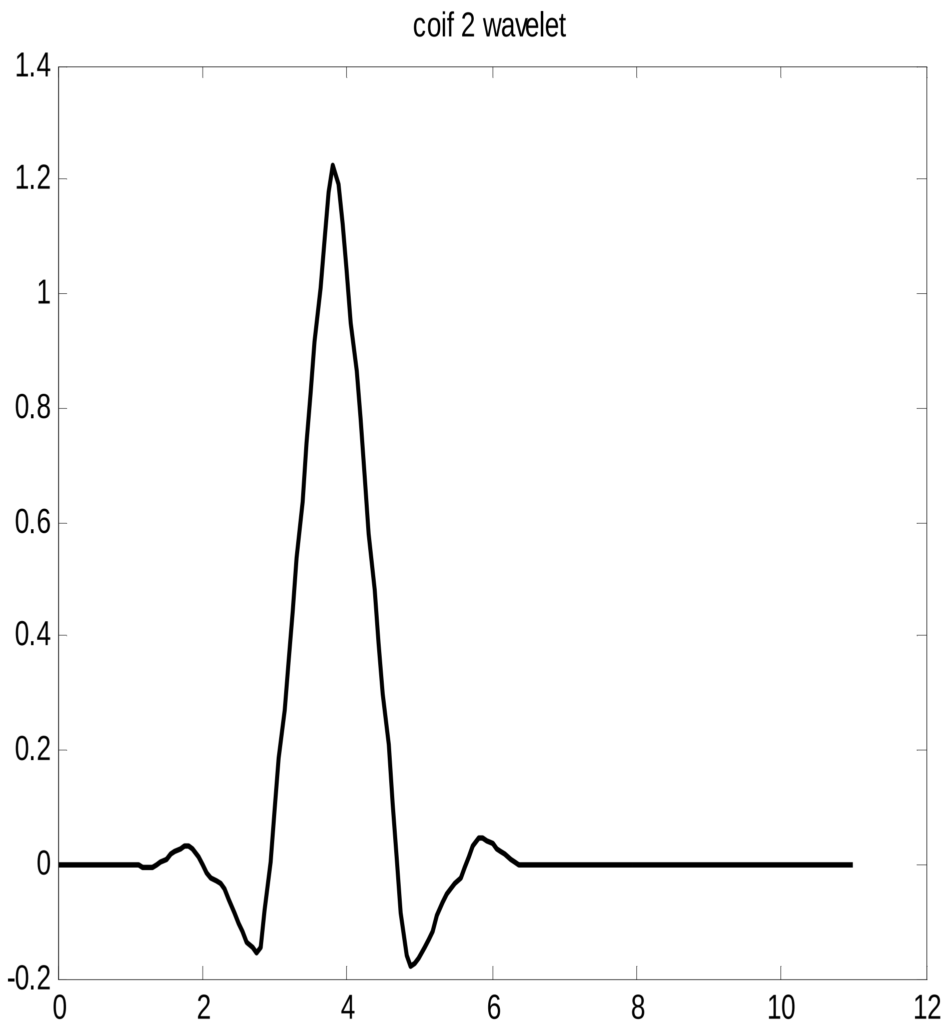
Figure(1.6): Daubechies wavelets for 6 iteration



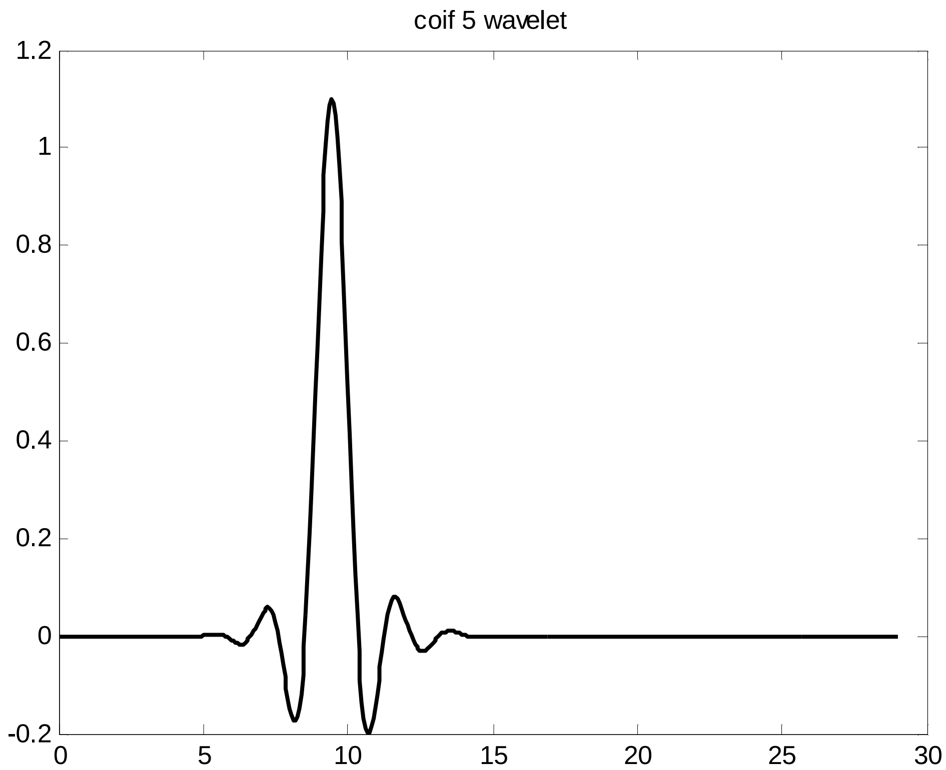
Figure(1.7): Different Types of Daubechies Wavelets Family

1.4.3. Coiflets Wavelet

Coiflets are discrete wavelets designed by Daubechies. Coiflet wavelets are near symmetric, their wavelet function $N/_3$, vanishing moments and scaling function$N/_3 - 1$. Both the scaling function and wavelet function can be normalized by a factor $1/\sqrt{2}$. Coiflet wavelets are orthogonal compactly supported wavelets with the highest number of vanishing moments for wavelet function for given support width. The Coiflet wavelets are more symmetric and have more vanishing moments than Daubechies wavelets. The Coiflet wavelet function with order 2 and 5 shows respectively figure(1.8) and figure (1.9).



Figure(1.8): Coiflet wavelet function with order 2



Figure(1.9): Coiflet wavelet function with order 5

1.4.4. Symlets wavelets

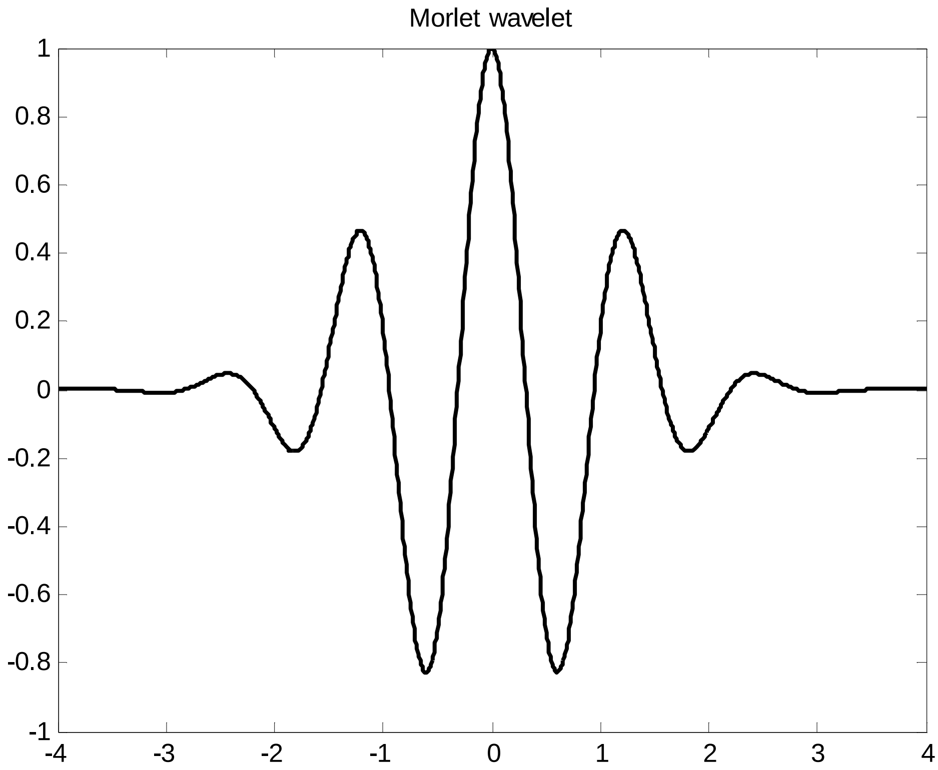
The properties of symlets wavelets are near symmetric, orthogonal and biorthogonal. Symlets wavelet families are Symlets 2 to Symlets 20. Symlets wavelets' are commonly used for removal of Gaussian additive noise from speech signal. Symlets wavelets are proposed by Daubechies as modification to the Daubechies family. The properties of the two wavelet families are similar.

1.4.5. Morlet wavelets

The morlet wavelet is defined as:

$$\Psi(x) = Ce^{x^2} cos(5x) \tag{1.2}$$

The constant C was used for normalization in view of reconstruction. The support width of this wavelets is infinite. The effective width of Morlet wavelets is [-4,4]. The morlet wavelet functions are shown in figure (1.10)

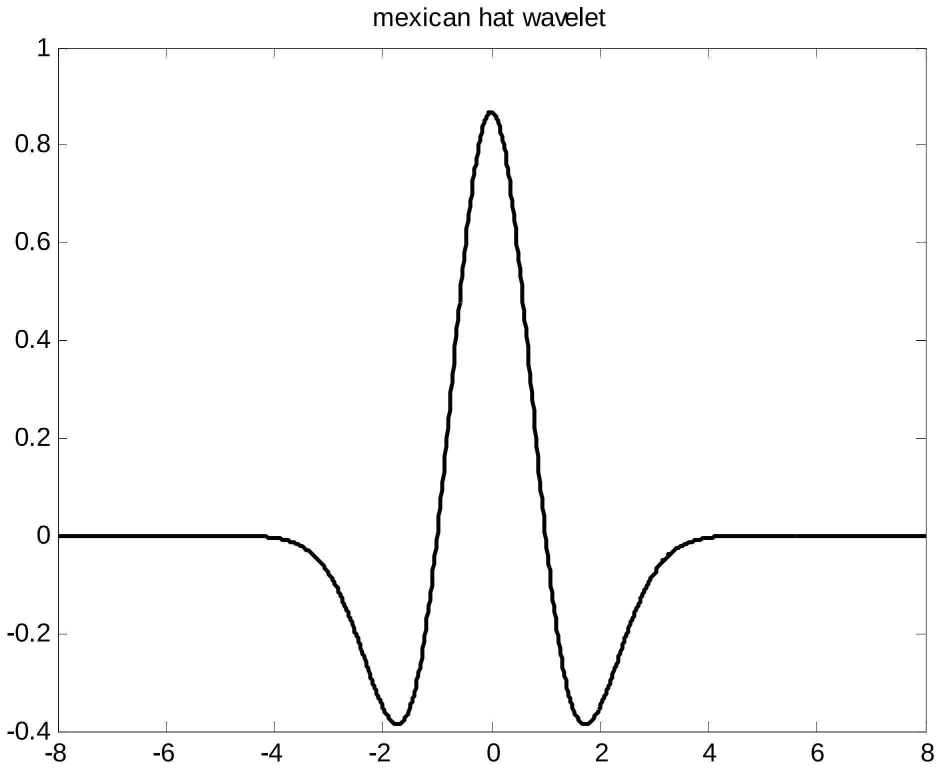


Figure(1.10): Morlet Wavelet Function

1.4.6. Mexican hat wavelet.

Mexican hat wavelet is proportional to the second derivative function of the Gaussian probability density functions. The wavelet is a special case of a larger family of derivative of Gaussian wavelets. There is no scaling function associated with this type of wavelet. The support width of these wavelets is infinite. The effective width of Mexican hat wavelets is [-5,5]. The Mexican hat wavelet shown in figure (1.11). The Mexican hat wavelet can be defined as:

$$\Psi(x) = \left(\frac{2}{\sqrt{3}}\pi^{-1/4}\right)(1 - x^2)e^{-x^2/2} \tag{1.3}$$



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Figure(1.11): Mexican Hat Wavelet

1.4.7. Biorthogonal wavelets

Biorthogonal family of wavelets exhibits the property of linear phase, which is used for signal and image reconstruction. Two types of wavelets are used; one for decomposition (on the left side) and the other for reconstruction (on the right side) instead of the same single one, interesting properties are derived. He order of the these wavelets are Nr and Nd. Nr for reconstruction and Nd for decomposition. The possible values for Nr and Nd are shown in table (1.1).

Table (1.1): Values of Nr And Nd for Biorthogonal Wavelets

Nr	Nd
1	1 , 3 or 5
2	2,4,6,or 8
3	1,3,5,7,or 9
4	4
5	5
6	8

1.5. INTRODUCTION TO WAVELET GALERKIN METHOD

The Wavelet Galerkin method is a powerful tool for solving partial differential equations.Wavelet Galerkin technique is the most frequently used in these days. Daubechies wavelet as bases in a Galerkin method to solve differential equations requires a computational domain of simple shape. The wavelet method has been shown to be a powerful numerical tool for the fast and accurate solution of differential equations.The connection coefficients plays and important role in applying wavelet galerkin method. Daubechies wavelets have been successfully used as a base function in Wavelet Galerkin method, due to their compact support, orthogonality and Multiresolution properties. The connection coefficients are employed in the calculation of stiffness, mass and geometry matrices. The one of the most possible applications of wavelet theory is numerical solution of differential equations. The Wavelet Galerkin method is applied to solve differential equations for structures like beam, column, plates etc. wavelet galerkin method require different types of connection coefficients for different differential or integral equations.

CHAPTER 2

REVIEW OF LITERATURE

2.1. INTRODUCTION

This chapter reviews the most relevant literatures on the wavelet Galerkin finite element method. Over the last two decades wavelets are being effectively used for signal processing and solution of differential equations. Wavelets have several properties which are encouraging their use for numerical solutions of partial differential equations (PDEs). The Wavelet-Galerkin method is a powerful tool for solving partial differential equations. The orthogonal, compactly supported wavelet basis of Daubechies exactly approximates polynomial of increasingly higher order. These wavelet bases can provide an accurate and stable representation of differential operations even in region of strong gradients or oscillations. In addition, the orthogonal wavelet bases have the inherent advantage of multi resolution analysis over the traditional methods. Some of the pertinent studies done recently are reviewed elaborately and critically discussed to identify the lacunae in the existing literature. The studies in this chapter are grouped into two major parts as follows

- Review of literature on wavelets method
- Review of literature on wavelet-Galerkin method

2.2. REVIEW OF LITERATURE ON WAVELETS METHOD

Wavelets are mathematical function that cut data into different frequency component. Wavelets methods are more advantageous than Fourier methods. Wavelets are developed independently in the field of mathematics, quantum physics etc.

Chui [9] presented the history of wavelets beginning with Fourier, compare wavelet transform with Fourier transform, state properties and other special aspects of wavelets and finishing with some interesting applications. This book gives some similarities and difference between wavelet transform and fourier transform

Daubechies [10] studied the construction of orthonormal bases of compactly supported wavelets with arbitrarily very high regularity. The order of regularity increases linearly with the support width. They start by reviewing the concept of Multiresolution analysis as well as several algorithms in vision decomposition and reconstruction. Then the construction follows from a synthesis of different approaches.

The exact and explicit representation of differential operators, in orthonormal bases of compactly support wavelets, is explained in Beylkin [5]. The method of computing this representation is directly applicable to multidimensional convolution operators. This describes a brief idea about wavelets, Hilbert transform, pseudo differential, shift operators, fractional derivatives and also describes numerical algorithms for finding connection coefficients.

Amaratunga and Williams [2] developed the use of wavelet basis function in solving partial differential equations. Wavelet theory provides various basis functions and multi-resolution methods for finite element method. In this paper, a wavelet-based beam element is constructed by using Daubechies scaling functions as an interpolating function. Since the nodal lateral displacements and rotations are used as element degrees of freedom, the connection between neighboring elements and boundary conditions can be processed simply as done for traditional elements.

Goedecker [35] presented the wavelets and their applications of partial differential equation in various field. This article is intended to make the theory of wavelets understandable to the audience. In addition to a self-contained and intuitive presentation of the theory of wavelets, extensive tables with the basic filter coefficients of differential operators in several wavelet families can be found in this article.

Kozbial [36] presented a new wavelet-based approach for solving two-dimensional boundary-value mechanical problems on the example of plate bending. The deflection equation of a bending plate is approximated by two-dimensional Daubechies wavelets using a least-squares Galerkin method. Due to the order of the differential equation in mechanics of plate structures is four, a way to perform the calculations of high order connection coefficients (that is, integrals of products of basis functions with their high order derivatives) is suggested. The implementation of two-dimensional Daubechies scaling functions approximation to plate bending is exhibited numerically in some examples. The results show that this method has good precision and reliability.

Amaratunga and Williams [3], have described how wavelets may be used for the temporal discretization of ordinary differential equation and partial differential equations. The first problem associated with the use of wavelets in is that initial conditions are difficult to impose. They are developed a method for solving initial value problem using wavelet extrapolation and they are demstrated the usage of the method for a linear mode problem.

Wave propagation deal with loadings that have very high frequency content. Hence the element size has to be comparable to wavelengths, which are very small at high frequencies. Fourier methods for frequency domain used for solving this type of problem, which can in principle achieve high accuracy in numerical differentiation. One such method is spectral finite element method (SFEM) developed by Doyle [15].

Mira and Gopalakrishnan [27] studied the spectral finite element modeling. Exact shape functions are derived and finite element procedure is followed in the transformed frequency domain. Here spectral finite element formulation is done using Daubechies scaling function bases for temporal approximation. In comparison to the conventional Fourier transform based spectral finite element method, the use of localized bases functions in the Daubechies scaling function based spectral finite element method allows accurate wave propagation analysis of finite length structures. First, numerical experiments are performed with narrow banded modulated pulse loading to obtain the location of damage from wave arrival time. Next, a broad banded impulse load is considered and effects of parameters like damage width, depth, and location on the responses are studied in time and frequency domains. WSFE is formulated for a notched Euler–Bernoulli beam. The effect of the notch is introduced in the formulation using perturbation technique. It should be mentioned here that as the analysis is restricted to damages with much smaller dimension, compared to that of the beam, the mode conversion resulting in axial and flexural coupling is not considered in the formulation. The present method is developed for pure flexural wave propagation and the effect of axial coupling due to the presence of the notch is neglected.

2.3. REVIEW OF LITERATURE ON WAVELET-GALERKIN METHOD

Wavelet Galerkin method is a powerful tool for solving partial differential equation. Wavelet galerkin method is the one of the best method for finding numerical solution of partial differential equation. The wavelet-Galerkin method providing an improvement over other methods by using compactly support wavelets.

Deka and Choudary [13] studied some special type of integrals of Daubechies wavelets, which are used as galerkin basis function for numerical solution of partial differential equation. Numerical and theoretical results are obtained for elliptical problem for second order with different types of boundary conditions. Instead of scaling functions, wavelet functions can also be used for this problem. Finally compare this result with finite difference method shows that the wavelet method is right competator of the classical methods.

Dahmen and Micchelli [12] developed the Wavelet Galerkin method used for solving partial differential equations leads to problem of computing integrals of products of derivatives of wavelets. In this paper studies the problem from point of view of stationary subdivision scheme. One of the main results is to identify these integrals as components of the unique solution of a certain eigenvector moment problem associated with the coefficient of refined equation.

Amaratunga and Williams [1] studied review of application of wavelet into the solution of partial differential equation. The theory of wavelets is described here using the language and mathematics of signal processing. This paper shows a method of adapting wavelets to an interval using an extrapolation technique called wavelet extrapolation. Wavelet extrapolation can be regarded as a solution to the problem of wavelets on an interval. In wavelet extrapolation technique a polynomial of p-1, p generally taken as the vanishing moments are assumed to be extrapolate the values at the boundaries.

Chen and Hawang [7] have described an exact evaluation of various finite integrals whose integrands involves product of Daubechies compactly support wavelets and their derivatives and integrals. These finite integral plays an important role in the wavelet-Galerkin approximation of differential or integral equations on bounded interval.

Besora [21] developed Galerkin wavelet method for global waves in 1D. Galerkin wavelet method for solving partial differential equations has been tested for two particular cases. The results of the first case, a harmonic wave equation. The results of the second case, a biharmonic wave equation, show that a more accurate method for the solution of wavelet overlap integrals, involving more than two basis functions are required.

Jin and chang [20] presented wavelet function apply to the numerical solution of differential equation. Antiderivatives of wavelets are used for numerical solution for differential equation. The orthogonal property of the wavelet is used to construct efficient interactive methods for the solution of the resultant linear algebraic system. And also give some numerical examples. Optimal error estimates are got in the application of two point boundary value problem of second order differential equation.

Jin and Ye [16] presented the compactly supported wavelet- based numerical solution of boundary value problem This problem can be discretized by the wavelet-Galerkin method. The evaluation of connection coefficients plays an important role in applying wavelet galerkin method to solve partial differential equations.

Santos and Burgos [33] presented the use of compactly support wavelet functions has become increasingly popular in the development of numerical solution of partial differential equations, especially for problems with local high gradient. He presented the formulation and validation of the Wavelet Galerkin method using Deslauriers-Dubuc interpolates. It was also shown that the wavelets have the ability of capturing the discontinuities without the need to place nodes where they occur.

Vinod and Sabina [37] presented the wavelet techniques and apply Galerkin procedure to analyze one dimensional harmonic wave equation as a test problem using fictitious boundary approach. Solutions of differential equation obtained using the Daubechies 6, 8 and 12 coefficients wavelets have been compared with the exact solution.

2.2.1. Review of Literature on Connection Coefficients

The connection coefficient is an integral of products of wavelets basis function their derivative and translation. Latto, Resnikoff and Tenenbaum [23], have described an exact method for evaluating connection coefficients. It is essential for the application of wavelets to the numerical solution of partial differential equations.

Romie and Peyton [32], have shown proper connection coefficients for compactly support wavelets. Proper connection coefficients are important for the solution of non periodic partial differential equation. They have demonstrated a technique for deriving a linear system whose solution in the set of proper connection coefficients needed to compute the natural inner product on bounded interval. They exhibit a simple one dimensional test problem that illustrated the use of proper connection coefficients for partial differential equations on bounded domains with dirchilet boundary condition.

Popovici [28], dealt with solutions to partial differential equations or ordinary differential equations using the wavelet galerkin method. For finding the connection coefficients several algorithms have been worked out and MATLAB programs. In those papers they described the MATLAB evaluation of connection coefficient.

Latto [23] described a connection coefficient is an integral of products of wavelet basis function, their derivatives and translates and also give the exact method for calculating the connection coefficients. This is essential for the application of wavelet to the numerical solution of partial differential equations. This gives the solutions of three term and two term connection coefficients. A Fortran code written by Restrepo [31] exists to compute 2-term and 3-term connection coefficients using periodized wavelets.

CHAPTER 3

PROPETIES OF WAVELETS

3.1 INTRODUCTION

The discrete wavelet transforms are developed as a special family of wavelet functions.This type of wavelets are compactly supported, orthogonal or biorthogonal and are characterized by low pass and high pass analysisand synthesis filters.. The orthonormal basis of compactly supported wavelet is generated by dilation and translation of wavelet function. The complete basis of wavelet can be formed through dilation and translation of mother wavelet scaling function. The wavelet consists of two functions, the scaling function and wavelet function. The scaling function describes the low pass filter for the wavelet transform and the wavelet function describes the band pass filter for the wavelet transform.

3.2 DAUBECHIES COMPACTLY SUPPORTED WAVELET

The family of this type of wavelet constructed by Daubechies includes members from highly localized to highly smooth.The orthonormal basis of compactly supported wavelet is generated by dilation and translation of wavelet function.The regularity order increases linearly with the support condition. All the examples of compactly supported wavelet bases are conspicuously non symmetric in contrast to infinitely support wavelet bases. Compactly supported wavelets have been successfully applied in numerical simulation.The higher order Daubechies function are not easy to describe with an analytical expression. The order of Daubechies functions are denoted by the vanishing moments.

3.3 WAVELET FUNCTION

Thewavelet function describes the band pass filter for the wavelet transform. The wavelet function is also called mother wavelets. The wavelet function can be defined as

$$\Psi(x) = \sum_{k=2-N}^{1} (-1)^k a_k \, \varphi(2x - k) \qquad (3.1)$$

The fundamental support of the scaling function $\varphi(x)$ is in the interval [0. N-1]

Figure (3.1) shows different families of Daubechies wavelets. The wavelet coefficients are calculated by reversing the order of the filter coefficients or coefficients of scaling function and reversing the sign of every second one. Mathematically, wavelet coefficients are expressed as:

$$b_k = (-1)^k a_{N-1-k} \qquad (3.2)$$

The Daubechies wavelet coefficients are presented in table (3.1)

Table 3.1 : Daubechies Wavelet coefficients $b_k$for N= 4, 6,8,10

K	N=4	N=6	N=8	N=10
0	-0.18301	0.0498	-0.0150	0.0047
1	-0.31698	0.1208	-0.0465	0.0178
2	1.18301	-0.1909	0.0436	-0.0088
3	-0.683012	-0.6504	0.2645	-0.1097
4		1.1411	-0.0396	-0.0456
5		-0.4705	-0.8922	0.3427
6			1.0109	0.1958
7			-0.3258	-1.0243
8				0.8539
9				-0.2264

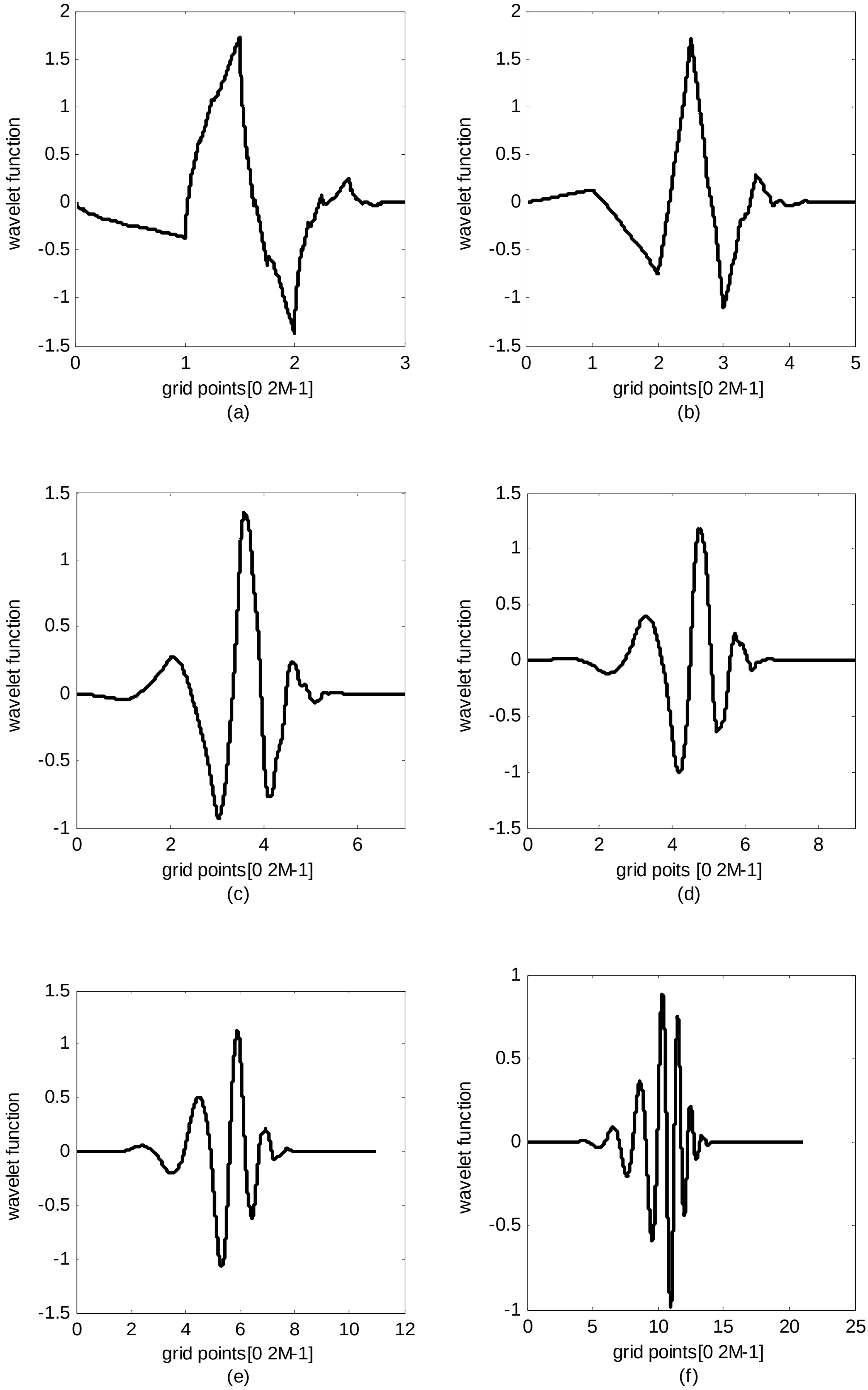


Figure 3.1: Wavelet function for different families of Daubechies wavelet:

3.4 SCALING FUNCTION

The scaling function describes the low pass filter for the wavelet transform. In the following expression, known as the two-scale relation, a_k are the filter coefficient of the wavelet scale function φ and N is the wavelet order.

$$\varphi(x) = \sum_{k=0}^{N-1} a_k \, \varphi(2x - k) \tag{3.3}$$

The fundamental support of the scaling function $\varphi(x)$ is in the interval [0. N-1] .the coefficients $a_k$in the above relation (3.3) is called wavelet filter coefficients.

The scaling functions have the following properties

$$\int_{-\infty}^{\infty} \varphi(x) = 1 \tag{3.4}$$

$$\int_{-\infty}^{\infty} \varphi(x - j)\, \varphi(x - m) dx = \delta_{j,m} \tag{3.5}$$

$$\int_{-\infty}^{\infty} x^k \, \psi(x) dx = 0 \tag{3.6}$$

The scaling functions are obtained by solving recursively the dilation equation(3.3) which can be expanded as

$$\varphi(x) = a_0\varphi(2x) + a_1\varphi(2x - 1) + \cdots + a_{N-1}\varphi(2x - N + 1) \tag{3.7}$$

$$\varphi(0) = a_0\varphi(0) \tag{3.8}$$

$$\varphi(1) = a_0\varphi(2) + a_1\varphi(1) + a_2\varphi(0) \tag{3.9}$$

$$\varphi(2) = a_0\varphi(4) + a_1\varphi(3) + a_2\varphi(2) + a_3\varphi(1) + a_4\varphi(0) \tag{3.10}$$

$$\varphi(N-2) = a_{N-3}\varphi(N-1) + a_{N-2}\varphi(N-2) + a_{N-1}\varphi(N-3) \tag{3.11}$$

$$\varphi(N-1) = a_{N-1}\varphi(N-1) \tag{3.12}$$

This can also be written as a matrix form

$$\begin{bmatrix} a_0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ a_2 & a_1 & a_0 & \cdots & 0 & 0 & 0 \\ a_4 & a_3 & a_2 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & a_{N-3} & a_{N-4} & a_{N-5} \\ 0 & 0 & 0 & \cdots & a_{N-1} & a_{N-2} & a_{N-3} \\ 0 & 0 & 0 & \cdots & 0 & 0 & a_{N-1} \end{bmatrix} \begin{bmatrix} \varphi(0) \\ \varphi(1) \\ \varphi(2) \\ \cdots \\ \varphi(N-3) \\ \varphi(N-2) \\ \varphi(N-1) \end{bmatrix} = \begin{bmatrix} \varphi(0) \\ \varphi(1) \\ \varphi(2) \\ \cdots \\ \varphi(N-3) \\ \varphi(N-2) \\ \varphi(N-1) \end{bmatrix} \tag{3.13}$$

i.e,

$$A\varphi = \varphi \tag{3.14}$$

Above equation indicates that the unknown vector φ is the eigenvector of the matrix [A]. As in all eigenvalue problems the solution to the system is not unique, and so a normalizing condition is required in order to determine a unique eigenvector.

The values of $\varphi(x)$ are known at the integer values of x between 0 to N-1 , the values at the points in between the integers can be obtained from the equation(3.3) modified as

$$\varphi\left(\frac{x}{2}\right) = \sum_{k=0}^{N-1} a_k\varphi(x-k) \tag{3.15}$$

Figure (3.2) shows Daubechies scaling function at for different orders. Scaling function corresponding to N=6 presented in table (3.3).

3.4.1 Filter coefficients

The filter coefficients can be calculated by using MATLAB wavelet tool box function *dbwavf*. The filter coefficients of order of Daubechies wavelet for 4, 6,8,10 are given in table(3.2). The filter coefficients satisfy the following conditions:

$$\sum_{k=0}^{N-1} a_k = 2 \tag{3.16}$$

$$\sum_{k=0}^{N-1} a_k a_{k-m} = 2\delta_{0,m} \tag{3.17}$$

$$\sum_{k=0}^{N-1} (-1)^k a_k . k^l = 0 \qquad l = 0,1,2, \dots \dots {}^N/_2 - 1 \tag{3.18}$$

$$\sum_{k=2-N}^{1} (-1)^k a_{1-k} a_{1-2m} = 0 \qquad l = 0,1,2, \dots \dots {}^N/_2 - 1 \tag{3.19}$$

Table 3.2 : Daubechies Wavelet filter coefficients $a_k$for N= 4, 6,8,10

K	N=4	N=6	N=8	N=10
0	-0.18301	0.0498	-0.0150	0.0047
1	-0.31698	0.1208	-0.0465	0.0178
2	1.18301	-0.1909	0.0436	-0.0088
3	-0.683012	-0.6504	0.2645	-0.1097
4		1.1411	-0.0396	-0.0456
5		-0.4705	-0.8922	0.3427
6			1.0109	0.1958
7			-0.3258	-1.0243
8				0.8539
9				-0.2264

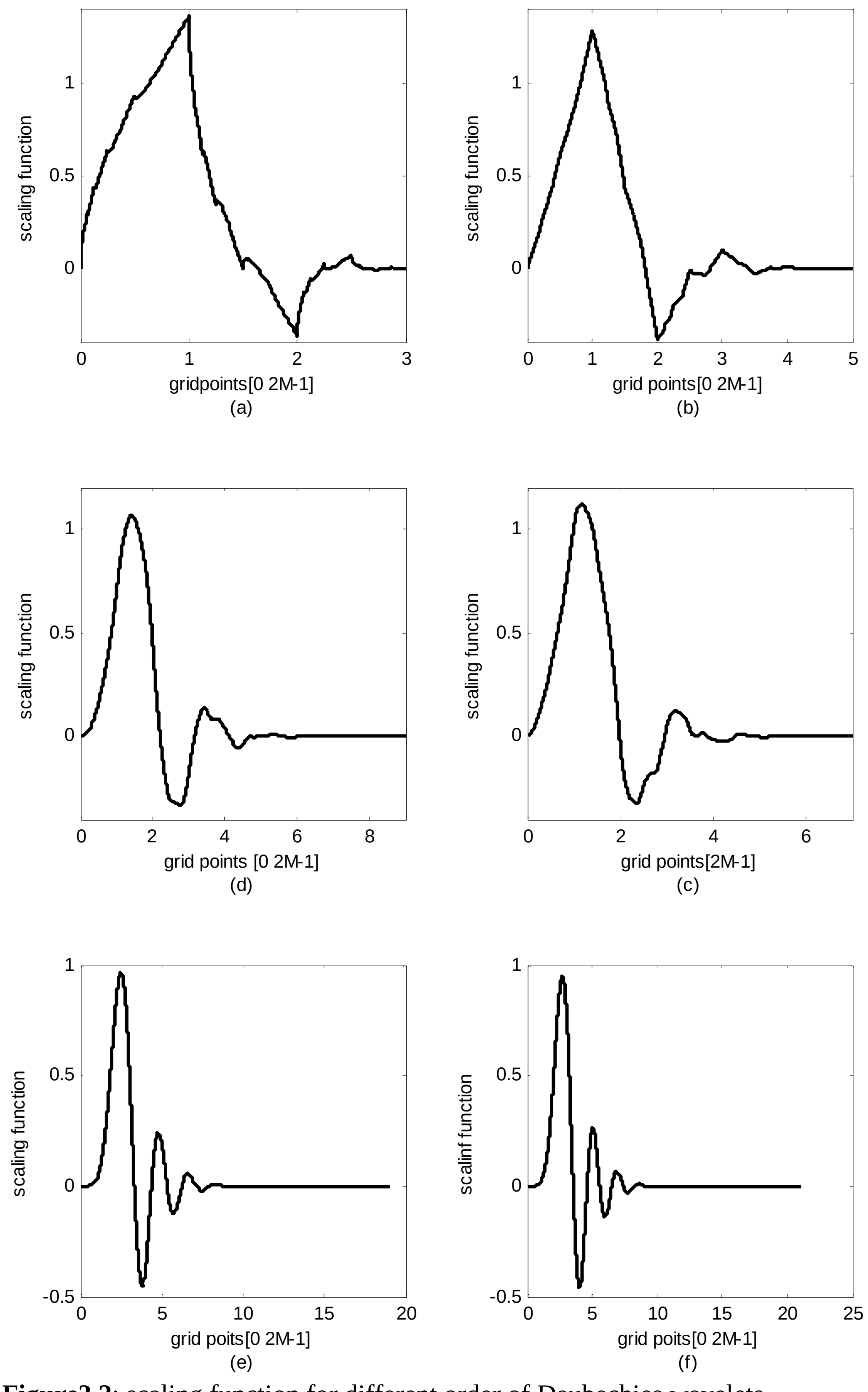


Figure3.2: scaling function for different order of Daubechies wavelets

Table (3.3): Scaling Functions

x	$\varphi(x)$
0	0
0.5	0.6505178
1	1.2863351
1.5	0.4411248
2	-0.385836
2.5	-0.014970
3	0.0952675
3.5	-0.031541
4	0.004234
4.5	0.002109
5	0

3.4.2 Derivatives of Scaling Function

As there is no analytical expression for wavelets, derivatives are obtained in dyadic grid points and the refinement of the solution depends on the level of resolution needed. The scale relation can be differentiated *n* times, generating the following expression:

$$\varphi^{n}(x) = \frac{d^{n}\varphi(x)}{dx} = \frac{d}{dx}\varphi^{n-1}(x) \tag{3.20}$$

Therefore,

$$\varphi^{(0)}(x) = \varphi(x) \tag{3.21}$$

Let $\varphi(x)$be the nth derivative of the scaling function, its denoted by:

$$\varphi^{n}(x) = 2^{n}\sum_{k=0}^{N-1} a_k\, \varphi^{n}(2x-k) \qquad (3.22)$$

Where n=0, 1, 2,…..N/2-1

The above relation is the two scale relation for $\varphi^{n}(x)$. This two scale relation can be used to find the values of $\varphi^{n}(x)$ for all dyatic points. To find out the values of $\varphi^{n}(x)$at integer the values of x are substituted from 1 to N-2 in equation (3.22. The homogeneous linear system of equationsobtained as

$$2^{-n}\varphi = A\varphi \qquad (3.23)$$

Where

$$\varphi = [\varphi^{n}(1)\varphi^{n}(2)\varphi^{n}(3)\ldots\ldots\ldots\varphi^{n}(N-2)] \qquad (3.24)$$

$$A = [a_{2i-k}]_{1\le i,k\le N-2} \qquad (3.25)$$

In the equation(3.23) the unknown vector φ is the eigenvector of the matrix [A], it corresponding to the eigenvalue2^{-n} . For all eigenvalue problems, the solution to the system was not unique. The values of φ can be determined uniquely by first finding the eigenvector of the matrix [A] corresponding to the eigenvalue 2^{-n}, and then normalizing with the condition

$$\sum_{k=1}^{N-2}(-1)^{n}\varphi^{n}(k) = n! \qquad (3.26)$$

For n=0, equation (3.22) is similar to the two scale relation (3.3) for $\varphi(x)$. Table (3.4) give the values of first, second and third order of scaling function corresponding to N=6. Figure (3.3) and figure (3.4) shows the first and second derivatives of scaling functions for N=6.

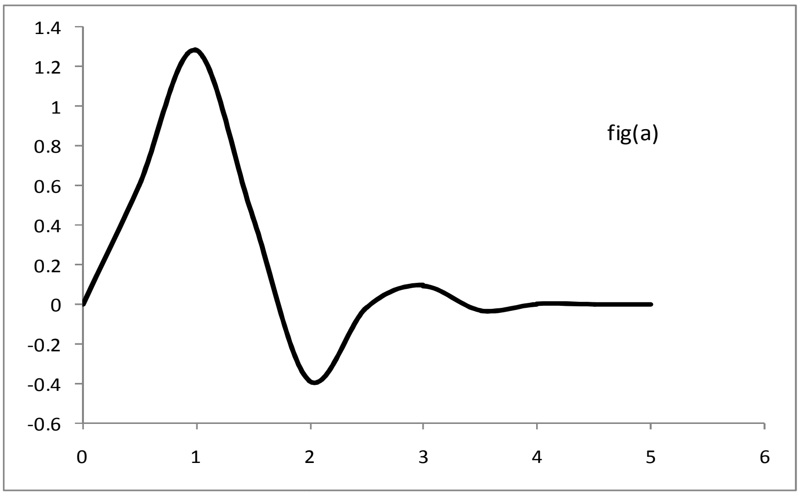


Figure3.3: first derivative of scaling functions for N=6

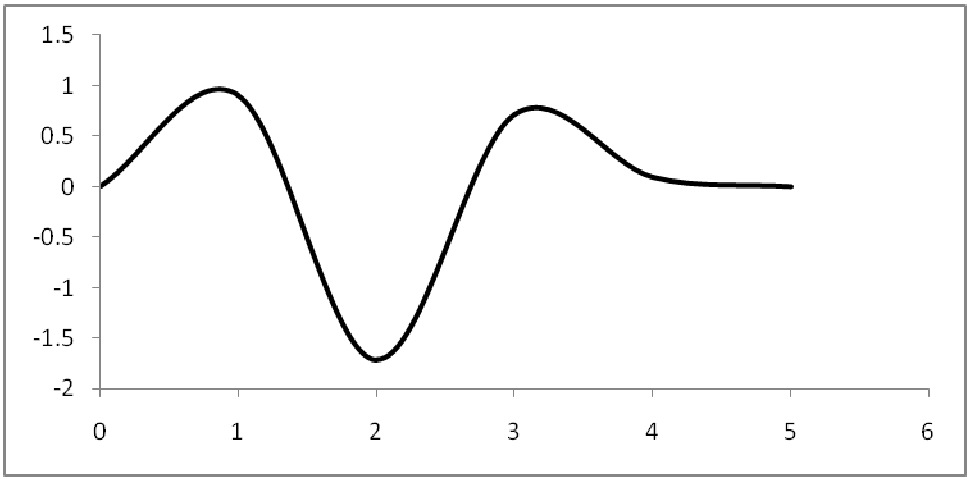


Figure3.4: second derivative of scaling functions for N=6

3.4.3 Evaluation of Integral of Scaling Function

Define the integaral of scaling function, $\varphi(x)$as $\gamma(x)$. The n-tuple integrals of $\varphi(x)$ denoted by $\gamma_n(x)$, i.e,

$$\gamma_n(x) = \int_0^x \gamma_{n-1}(y)dy \tag{3.27}$$

$$\gamma_n(x) = 2^{-n} \sum_{k=0}^{N-1} a_k \gamma_n(2x-k) \tag{3.28}$$

$$\gamma_1(x) = \int_0^{N-1} \varphi(y)dy + \int_{N-1}^{x} \varphi(y)dy = 1 \tag{3.29}$$

$$\gamma_2(x) = \int_0^{N-1} \gamma_1(y)dy + \int_{N-1}^{x} \gamma_1(y)dy \tag{3.30}$$

$$= \gamma_2(L-1) + \int_{N-1}^{x} 1dy$$

$$= (L-1) + (x-N+1) \tag{3.31}$$

$$\gamma_3(x) = \int_0^{N-1} \gamma_2(y)dy + \int_{N-1}^{x} [\, \gamma_1(N-1) + (y-n+1)]\, dy \tag{3.32}$$

$$= \gamma_3(N-1) + \gamma_2(N-1)(x-N+1) + \frac{1}{2!}\gamma_1(N-1)(x-N+1)^{\,2}$$

From the above equation, we can $\gamma_n(x)$ for $x \geq N-1$ as follows:

$$\gamma_n(x) = \sum_{\mathrm{i}=0}^{\mathrm{n}-1} \frac{(\mathrm{x}-\mathrm{N}+1)^{\mathrm{j}}}{\mathrm{i}!} \gamma_{\mathrm{n}-\mathrm{i}}(\mathrm{N}-1)$$

The value of $\gamma_1(N-1) = 1$, butthe values of $\gamma_{\mathrm{n}-\mathrm{i}}(\mathrm{N}-1)$ for i= 0,1,2,….n-2 are to be still determined. For determine $\gamma_1(N-1)$ for n=2,3,…, we go back to equation (3.27)

$$\gamma_n(N-1) = 2^{-n} \sum_{k=0}^{N-1} a_k \gamma_n(2N-2-k) \tag{3.34}$$

From equation(3.31), $\gamma_n(N-1)$ can be expressed as

$$\gamma_n(N-1) = 2^{-n}\sum_{k=0}^{N-1} a_k \sum_{i=0}^{n-1} \frac{(N-1-k)^j}{i!}\gamma_{n-i}(N-1) \qquad (3.35)$$

$$= 2^{-n}\left[\gamma_n(N-1)\sum_{k=0}^{N-1} a_k + \sum_{i=1}^{n-1}\sum_{k=0}^{N-1} a_k \ \frac{(N-1-k)^i}{i!}\gamma_{n-i}(N-1)\right]$$

From the relation(3.15) , we then have the recursive formula

$$\gamma_n(N-1) = \frac{1}{2^{n-2}}\sum_{i=1}^{n-1}\left(\sum_{k=0}^{N-1} a_k \frac{(N-1-k)^j}{j!}\right)\gamma_{n-i}(N-1) \qquad (3.36)$$

The values of $\gamma_n(x)$ for x =1,2,….N-2 and can be determined from the following linear system of equations:

$$(I - 2^{-n}A)\gamma_n = B \qquad (3.37)$$

Where,

$$A = [a_{2i-k}]_{1\le i,k\le N-2} \qquad (3.38)$$

$$\gamma_n = [\gamma_n(1)\gamma_n(2) \quad \dots\dots \ \gamma_n(N-2)] \qquad (3.39)$$

$$B = [b_1 b_2 \ \dots\dots \ b_{N-2}] \qquad (3.40)$$

$$b_i = \sum_{\substack{2i-k\ge N-1 \\ k=0,1,\dots N-1}} a_k \gamma_n(2i-k) \qquad (3.41)$$

The values of integral of scaling function for n=1 is given in table (3.4).

Table 3.4: Derivatives and integrals of scaling function for N=6

x	$\varphi^{1}(x)$	$\varphi^{2}(x)$	$r(x)$
0	0	0	0
1	1.6385	0.9042	0.6007
2	-2.2328	-1.7127	1.0967
3	0.5502	0.7127	0.9854
4	0.0441	0.0958	0.9996
5	0	0	1

3.4.4 Moments of Scaling Function

The another important property of wavelet is calculating the moments of scaling function. The moments of scaling function m_k^l denotes the kth moment of scaling function $\varphi(x)$, with the initial condition $m_0^l = 1$ and expressed as

$$m_k^l = \int_{-\infty}^{\infty} x^k \varphi\,(\,x) \tag{3.43}$$

Applying the scaling relationship in the above equation, we get a recursive relation,

$$m_k^l = \frac{1}{2^{k+1}-2} \sum_{i=0}^{N-1} \sum_{j=1}^{k} \binom{k}{j} a_i i^j m_{k-j}^l \tag{3.44}$$

3.4.5 Interpolation of scaling function

The common characteristics of interpolating wavelets require that the scaling function satisfies the following conditions:

$$\varphi(k) = \delta_{0.k} \qquad , \begin{cases} 1 & k = 0 \\ 0 & \end{cases} \tag{3.45}$$

Figure(3.5) and figure(3.6) shows the interpolate IN6 and IN8 respectively. All expression used for calculation of scaling function, derivatives of scaling function, connection coefficients and moments of Daubechies wavelet can be used to interpolate. Due to correlation, the support [0 N-1] in the expression for Daubechies becomes [1-N N-1] for interpolation.

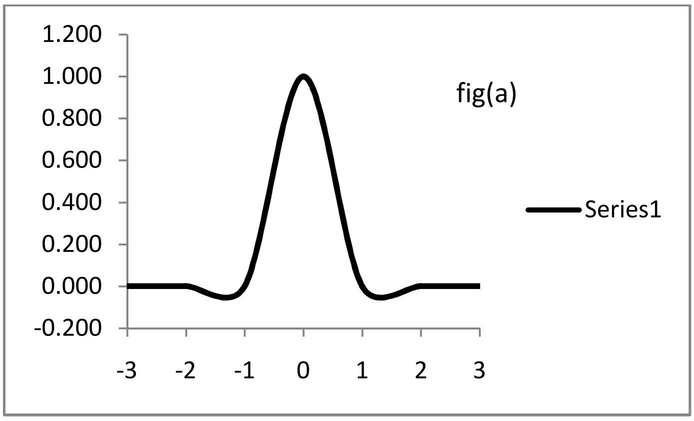


Figure (3.5): Interpolet IN6 Scaling Function

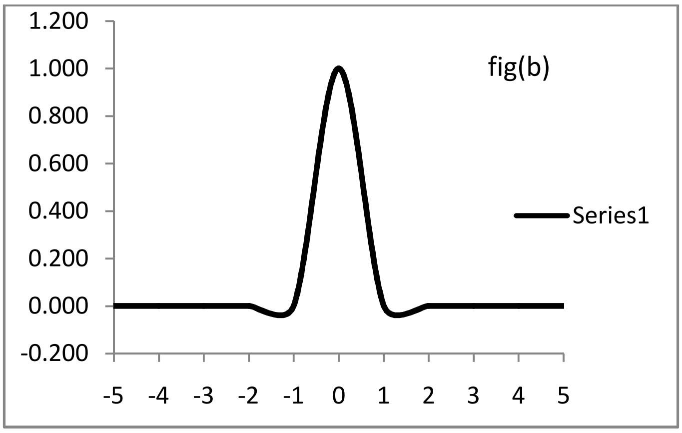


Figure (3.6): Interpolet IN8 Scaling Function

CHAPTER 4

CONNECTION COEFFICIENTS

4.1 INTRODUCTION

A connection coefficient is an integral of products of wavelet basis function, their derivatives and translates. Connection coefficients play an important role in wavelet galerkin method.. Connection coefficients are essential for application of wavelets to the numerical solution of partial differential equations, since numerical approximations of connection coefficients are in general unstable due to oscillatory nature of integrands. Here, we explain two term and three term connection coefficients.

4.2 CONNECTION COEFFICIENTS

The Wavelet-Galerkin scheme involves the evaluation of connection coefficients, to approximate derivatives as well as non-linear terms. The connection coefficients are the integrals with the integrands being product of wavelet bases and their derivatives. Owing to derivatives of compactly supported wavelet being highly oscillatory, it is difficult to compute the connection coefficients by the numerical evaluation of integrals. In order to overcome this problem, dedicated algorithms have been devised for the exact evaluation of connection coefficients. The connection coefficients and associated computation algorithms are essentially based on unbounded domain. Wavelet Galerkin method are limited to cases where the problem domain is unbounded or the boundary condition is periodic.

4.3 DETERMINATION OF CONNECTION COEFFICIENTS

4.3.1 Determination of two-term connection coefficients

The connection coefficients can be defined as the integral product of the scaling function $\varphi(x)$ and its n[th] derivative $\varphi^n(x-k)$ as

$$\Delta_k^n(x) = \int_0^x \varphi^n(y-k)\varphi(y)dy \tag{4.1}$$

The determination of above connection coefficient plays an important role in applying the wavelet-Galerkin method to solve differential equations. Substituting the equation (3.5) and (3.25) to equation (4.1), we have

$$\Delta_k^n(x) = \int_0^x \left[2^n \sum_{i=0}^{N-1} a_i \varphi^n(2y-2k-i)\right]\left[\sum_{j=0}^{N-1} a_j\, \varphi(2y-j)\right] dy \tag{4.2}$$

$$= 2^n \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} a_i a_j \int_0^x \varphi^n(2y-2k-i)\varphi(2y-j)dy$$

$$= 2^{n-1} \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} a_i a_j \int_0^x \varphi^n(2y-2k-i)\varphi(2y-j)d(2y-j)$$

$$= 2^{n-1} \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} a_i a_j \int_0^{2x-j} \varphi^n(\hat{y}-2k-i+j)\varphi(\hat{y})d\hat{y}$$

$$= 2^{n-1} \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} a_i a_j\, \Delta_{2k+i-j}^n(2x-j)(4.3)$$

Followings are the important properties of the two term connection coefficients:

$$when\ x \geq N-1 \quad \Delta_k^n(x) = \Delta_k^n(N-1)(4.4)$$

$$when\ |k| \geq N-1, x \leq 0\ or\ x \leq k \quad \Delta_k^n(x) = 0 \tag{4.5}$$

$$when\ k \geq 0 \quad \Delta_{-k}^n(N-1) = (-1)^n \Delta_k^n(N-1)(4.6)$$

$$when\ x-k \geq N-1 \quad \Delta_{-k}^{n}(N-1) = (-1)^n \Delta_k^n(N-1) \tag{4.7}$$

Evaluation of $\mathbf{\Delta_k^n(N-1)}$

To compute the values of $\Delta_k^n(N-1)$ for k=0,1,2,…..,N-2.

From equation (3.48), we have,

$$\Delta_k^n(N-1) \; = 2^{n-1} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} a_i a_j \, \Delta_{2k+i-j}^n (2N-2-j)$$

$$= 2^{n-1} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} a_i a_j \, \Delta_{2k+i-j}^n (N-1) \tag{4.8}$$

For n>0, the equation substituting k= 0,1,2…..N-2 into equation(3.49), we obtain following homogeneous linear system of equation.

$$2^{1-n} \Delta^n(N-1) = B\Delta^n(N-1) \tag{4.9}$$

where,

$$\Delta^n(N-1) = [\Delta_0^n(N-1)\Delta_1^n(N-1) \;\; \dots \dots \quad \Delta_{N-2}^n(N-1)]^T \tag{4.10}$$

$$B = [b_{l.m}]_{0 \leq l.m \leq N-2} \tag{4.11}$$

$$b_{l,m} = 2^{n-1} \left(\sum_{\substack{0 \leq i,j \leq N-1 \\ 2l+i-j=m}} a_i \, a_j + (-1)^n \sum_{\substack{0 \leq i,j \leq N-1 \\ 2l+i-j=-m}} a_i \, a_j \right) \tag{4.12}$$

In the formulation of equation (4.8) the property (4.4) to (4.7) has been used. From equation (4.9) the unknown vector $\Delta_k^n(N-1)$ is the eigen vector of the matrix [B] that corresponds to the unity eigenvalue. However, the homogeneous linear system of equation does not admit the unknown vector $\Delta_k^n(N-1)$ to be determined uniquely. Therefore an additional normalizing condition is required.

We have,

$$\sum_{-\infty}^{\infty} k^n \varphi^n(x-k) = n! \tag{4.13}$$

Multiplying both side of equation (3.58) by $\varphi(x)$ and integrating from -∞ to +∞.

$$\sum_{-\infty}^{\infty} k^n \int_{-\infty}^{\infty} \varphi^n(x-k)\varphi(x)dx = n! \int_{-\infty}^{\infty} \varphi(x)dx \tag{4.14}$$

where,

$$\sum_{-\infty}^{\infty} k^n \int_{-\infty}^{\infty} \varphi^n(x-k)\varphi(x)dx = \sum_{-\infty}^{\infty} k^n \Delta_k^n(N-1) \tag{4.15}$$

$$n! \int_{-\infty}^{\infty} \varphi(x)dx \ = n! \tag{4.16}$$

We obtain the normalizing condition as,

$$\sum_{1}^{N-2} k^n \Delta_k^n(N-1) = \frac{n!}{2} \tag{4.17}$$

The above equation (4.17) is desired normalizing condition for the eigenvector of the matrix [B] associated with the unity eigenvalue for n>0

Evaluation of $\Delta_{\mathbf{k}}^{\mathbf{n}}(\mathbf{x})$ for x=1,2,3,…, N-2

Using the properties (3.49) to (3.52) that there are only $(N-2)^2$ independent members.

Substituting k= x-N+2,…..x-1 and x=1,,3,….N-2 into equation (3.48), we obtain following linear system of equation:

$$(2^{1-n}I - C)\Delta^n = d \qquad (4.18)$$

where,

$$\Delta^n = [\Delta^n(1)\Delta^n(2)\ \ldots \quad \Delta^n(N-2)]^T \qquad (4.19)$$

$$\Delta^n(x) = [\Delta^n_{x-N+2}\Delta^n_{x-N+3} \quad \ldots\ldots \quad \Delta^n_{x-1}]^T \qquad (4.20)$$

$$C = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & \cdots & \cdots & c_{1,N-2} \\ c_{2,1} & c_{2,2} & \cdots & \cdots & \cdots & c_{2,N-2} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ c_{N-2,1} & c_{N-2,2} & \cdots & \cdots & \cdots & c_{N-2,N-2} \end{bmatrix} \qquad (4.21)$$

$$c_{k,l} = \left[c_{k,l,m,n}\right]_{1\le m,n\le N-2} \qquad (4.22)$$

$$c_{k,l,m,n} = a_{2k-l}a_{N-1-2k+m} \qquad (4.23)$$

$$d = [d(1) \quad d(2)\ \ldots\ldots \quad d(N-2)^2]^T \qquad (4.24)$$

$$d(nN-2n+m) = \sum_{k,l\in\mu(m,n,N)} a_k a_l \Delta^n_{2m+k\_l}(N-1) \qquad (4.25)$$

The $\mu(m,n,N)$ is given by:

$$\mu(m,n,N) = \{(k,l): (N-1 \le 2n-2m-k \ or \ N-1 \le 2n-l), 0 \le k,l \le N-1\} \qquad (4.26)$$

For n=0, the values of Δ^0 can be obtained by solving the equation (4.18) directly. When n>0, equation (4.18) is a linear dependent system and the corresponding rank deficiency is n. In order to eliminate the singularity of equation (4.19) additional relations are required for the members Δ^n.

$$\sum_{-\infty}^{\infty} k^n \varphi^n(x-k) = n! \tag{4.27}$$

Multiplying both side by $\varphi(y)$ and integrating from y=0 to y=x:

$$\int_0^x \sum_{-\infty}^{\infty} k^n \varphi^n(y-k)\varphi(y)dy = n! \int_0^x \varphi(y)dy \tag{4.28}$$

$$\sum_{k=-\infty}^{\infty} k^n \Delta_k^n(x) = n!\,\gamma(x) \tag{4.29}$$

When x =m the above equation can be written as :

$$\sum_{k=m-N+2}^{m-1} k^n \Delta_k^n(m) = n!\ \gamma(x) - \nabla \tag{4.30}$$

Where,

$$\nabla = (-1)^n \left[\sum_{2-N \le k \le m-N+1} \nabla_{-k}^n(N-1) \right] \tag{4.31}$$

Table (4.1) presents the values of connection coefficients for N=6.

Table(4.1): The Values of Connection Coefficients for n=1 and N=6

X	**k**	$\Lambda_k^n(x)$	**x**	**k**	$\Lambda_k^n(x)$
	-4	0.00034	4	-4	0.00034
1	-3	-0.00960		-3	0.014611
	-2	0.24682		-2	-0.14539
	-1	-1.06420		-1	0.745205
	0	0.82732		0	0.000008
				1	0.745285
	-4	0.000342		2	-0.14539
	-3	0.014611		3	0.015053
	-2	0.14376			
2	-1	-0.77113		-4	-0.00034
	0	0.074435		-3	-0.014611
	1	0.56789	5	-2	0.14539
				-1	-0.745205
	-4	0.00034		0	0
	-3	0.014611		1	0.00034
	-2	-0.14539		2	0.014611
3	-1	-0.74488		3	-0.14539
	0	0.00453		4	0.745205
	1	0.73437			
	2	-0.12428			

4.3.2 Three-term connection coefficients

The integral is defined as:

$$\Omega_{j,k}^{m,n}(x) = \int_0^x \varphi(y)\varphi^m(y-j)\varphi^n(y-k)dy \tag{4.32}$$

The three -term connection coefficients play an important role in wavelet galerkin method.

$\Omega_{j,k}^{m,n}(x)$have the following properties:

$$\Omega_{j,k}^{m,n}(x) = 0 \quad for\ |j|, |k|, or\ |j-k| \geq N-1 \tag{4.33}$$

$$\Omega_{j,k}^{m,n}(x) = 0 \quad for \;\; x - j, x - k \; or \; x \leq 0 \tag{4.34}$$

$$\Omega_{j,k}^{m,n}(x) = \Omega_{j,k}^{m,n}(N-1) \;\; for \;\; x - j, x - k \; or \; x \geq N - 1 \tag{4.35}$$

The properties (4.33) and (4.34) comes from the fact that supports of$\varphi^{i}(x)$ for i= 0,1,2,3,…. are all in the interval [0, N-1]. The two scale relations are,

$$\varphi(x) = \sum_{k=0}^{N-1} a_k \, \varphi(2x - k) \tag{4.36}$$

$$\varphi^{n}(x) = 2^{n} \sum_{k=0}^{N-1} a_k \, \varphi^{n}(2x - k) \tag{4.37}$$

Substituting two scale relation (4.36) and (4.37) into equation (4.32).

$$\Omega_{j,k}^{m,n}(x) = 2^{m+n-1} \sum_{p=0}^{N-1} \sum_{q=0}^{N-1} \sum_{r=0}^{N-1} a_p \, a_q a_r \, \Omega_{2j+q-p,2k+r-p}^{m,n}(2x - p) \tag{4.38}$$

The values of $\Omega_{j,k}^{m,n}(N-1)$ for integers j,k,m,n can be computed. For this purpose from the properties (4.33) to (4.35), for fixed integers m and n, there are $3N^2 - 9N + 7$ unknown $\Omega_{\mathrm{j,k}}^{\mathrm{m,n}}(\mathrm{N}-1)$. The equation (4.38) associated with these unknown coefficients to form a linear homogeneous equation as follows:

$$R = 2^{1-m-n} SR \tag{4.39}$$

Where,

$$R = [R_{2-N} R_{3-N} \ldots\ldots\ldots \; R_{N-2}]^T \tag{4.40}$$

$$R_{j} = \left[\Omega^{\mathrm{m,n}}_{\mathrm{j},\alpha}(\mathrm{N}-1)\Omega^{\mathrm{m,n}}_{\mathrm{j},\alpha+1}(\mathrm{N}-1\;)\ldots\ldots\ldots\;\Omega^{\mathrm{m,n}}_{\mathrm{j},\mu}(\mathrm{N}-1)\right] \tag{4.41}$$

$\alpha = \max(j+2-N, 2-N)$ and $\mu = \min\,(j+N-2, N-2)$ and the elements of matrix [S] are sum of product of the form$a_p a_q a_r$. The matrix [S has the eigenvalue 2^{1-k} , k=0,1,2,…N-2 with their multiplicity order k+1. In equation (4.39) the vector R is the eigenvector of matrix [S] corresponding to the eigenvalue of 2^{1-m-n}. It is not sufficiently to determine the vector R uniquely from (4.39). For solving the unknown vector R, we need extra equations. The required extra equation is derived from the moment equation (4.27). First multiplying both sides of equation (4.27) by $\varphi^m(y-j)$ and $\varphi^n(y-k)$, respectively, and taking integration from y=0 to y=x, we have,

$$\sum_k k^n \Omega^{m,n}_{j,k}(x) = n!\Delta^m_j(x) \tag{4.42}$$

$$\sum_j j^m \Omega^{m,n}_{j,k}(x) = m!\Delta^n_k(x) \tag{4.43}$$

For fixed integers m and n , we can form an independent system of equations by eliminating the corresponding rows of the unknowns $\Omega^{m,n}_{j,0}, j = 2-N, 3-N, \ldots\ldots, 2-N+n,$ and of the unknowns $\Omega^{m,n}_{k,k}, k = 1,2,3, \ldots\ldots, m$. It is noted that such a replacement is rather difficult. In practice, however, the rank of the resultant system of equation can be checked by a numerical computation, and obtained the values of connection coefficients and verified by using the equation (4.42) and (4.43).

CHAPTER 5

NUMERICAL EXAMPLES

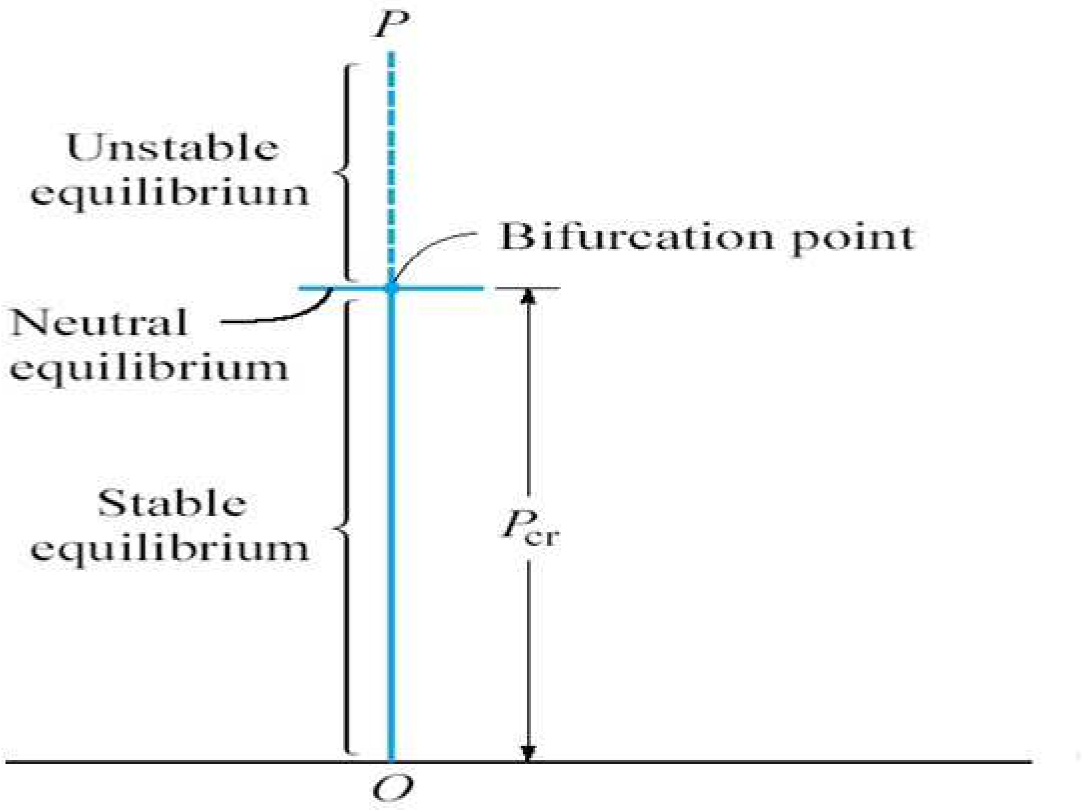
5.1. INTRODUCTION

The wavelet based numerical solution is recently used in the application of partial differential equation. The wavelet method is an efficient alternative of numerical solutions. In this chapter, the problems are discretized by wavelet galerkin method. The coefficient matrix of the wavelet methods are more difficult to calculate in practical application. The Daubechies wavelet has gained much interest in the solution of partial differential equations. The connection coefficients play an important role in applying wavelet galerkin method. The evaluation of connection coefficients are explained in chapter 3.

5.2. CRITICAL LOAD

Critical load was the only load for which the structure would be in equilibrium in the just disturbed position. If the axial load is very less than critical load the effect of the moment in the spring dominated and the structure returns to the vertical position. If the axial load of the member is larger than the critical load the effect of the axial force predominates and the structure buckles to unstable condition.

The boundary position between stability and instability is called the neutral equilibrium condition. At which the deflections of the member become very large, that critical point is called Bifurcation point of the system



5.3. WAVELET GALERKIN METHOD

The wavelet galerkin method is a powerful method for solving partial differential equations. Wavelet Galerkin technique is the most frequently used scheme these days. Daubechies wavelet as bases in a Galerkin method to solve differential equations requires a computational domain of simple shape. The wavelet method has been shown to be a powerful numerical tool for the fast and accurate solution of differential equations. The connection coefficients plays an important role in applying wavelet galerkin method. Daubechies wavelets have been successfully used as a base function in Wavelet Galerkin method, due to their compact support, orthogonality and Multiresolution properties. The connection coefficients are employed in the calculation of stiffness, mass and geometry matrices. The one of the most possible applications of wavelet theory is numerical solution of differential equations. The wavelet galerkin method can be applied to solve differential equations for structures like beam, column, plates etc. wavelet galerkin method require different types of connection coefficients for different differential or integral equation.

5.4. EXACT METHOD

Considering an ideal prismatic bar made of a linearly elastic material and subjected to an axial compressive load.

The governing equation is given by

$$EI\frac{d^2v}{dx^2} + Pv = 0 \tag{5.1}$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = 0 \tag{5.2}$$

This is a second order homogeneous ordinary differential equation with constant coefficients that has a solution of the form

$$v = Asinkx + Bcoskx \tag{5.3}$$

Boundary conditions for simply supported ends are

$$for\ x = 0\ ; \quad v = 0 \quad and \quad \frac{d^2v}{dx^2} = 0$$

$$for\ x = L\ ; \quad v = 0\ and \quad \frac{d^2v}{dx^2} = 0$$

Applying above boundary conditions to equation (5.3) we have;

B=0;

$$sinkL = 0$$

$$,\ kL = 0\ ,\ \pi\ , \quad 2\pi \ldots \ldots \ldots \ldots$$

$$k = \frac{n\pi}{L}$$

Buckling load must satisfy the relation $P = k^2 EI$

The critical load is taken as

$$P_{cr} = \frac{n^2\pi^2}{L^2} EI \tag{5.4}$$

for n=1

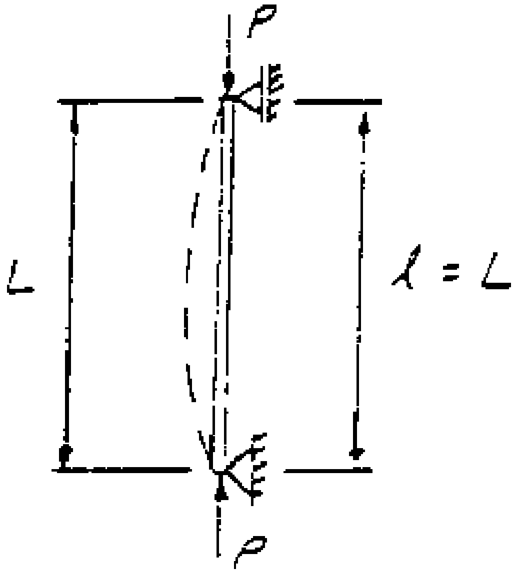
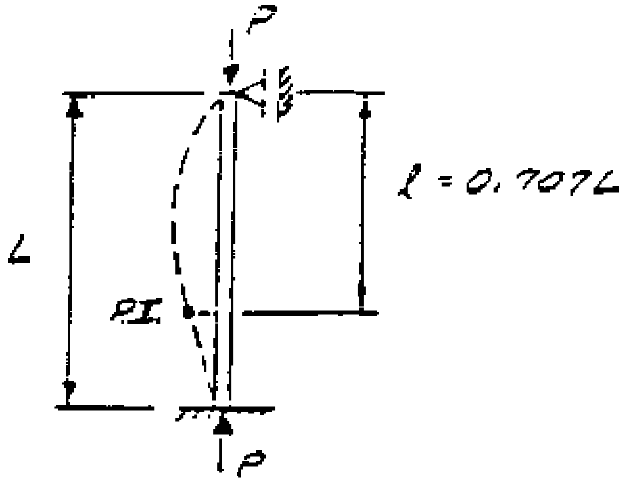
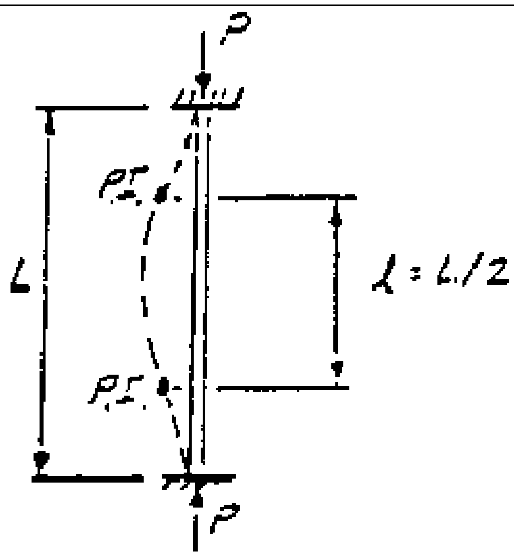
$$P_{cr} = \frac{\pi^2}{L^2} EI \tag{5.5}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{L^2 A} \tag{5.6}$$

The effective length of column is defined as the length of the column between point of inflection or point of zero moments . The effective length expressed in terms of total length by:

$$l = KL \tag{5.7}$$

where K is the "effective length factor" of the member depends on the support conditions. Effective length factors for different conditions are given in Figure (5.1)

Pinned - Pinned		***K* = 1.0**
Pinned - Fixed		**K = 0.707**
Fixed – Fixed		**K = 0.5**

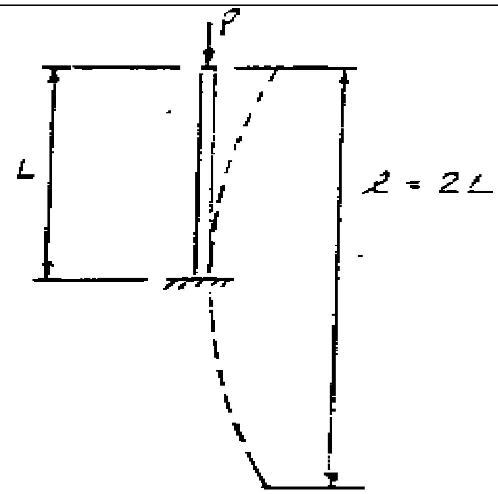
Cantilever		**K = 2**

Figure (5.1): Effective Length for Different Support Condition

5.5. PROBLEM FORMULATION

In this chapter solving the buckling problem of pressurized bar by using wavelet galerkin method. And demonstrate the effectiveness of the ‘connection coefficient method. The connection coefficient method explained in third chapter.

Consider an ideal prismatic bar of a linearly elastic material and subjected to an axial compressive load.

The bending differential equation of pressurized bar for buckling analysis is

$$EI\frac{d^2W(x)}{dx} + PW(x) = 0 \qquad (5.8)$$

$$EIW''(x) + PW(x) = 0 \qquad (5.9)$$

Where,

P = axial pressure on the bar

W = Difflection of the bar

L =length of the bar

E = modulus of elasticity of bar

I = moment of inertia of the bar

The equation (5.9) can be converted into its non dimensional form. The non dimensional form of equation is given by

$$\bar{W}''(\bar{X}) + P\bar{W}(\bar{X}) = 0 \tag{5.10}$$

The non dimensional parameters in the above equations are given by

$$\bar{W} = \frac{W}{L} \tag{5.11}$$

$$\bar{X} = \frac{X}{L} \tag{5.12}$$

$$\bar{P} = \frac{PL^2}{EI} \tag{5.13}$$

.

5.5.1. Clamped Support

The boundary condition for non dimensional clamped support is as follows

For x=0;

$$\bar{W}(0) = 0 \tag{5.14}$$

$$\bar{W}'(0) = 0 \tag{5.15}$$

For x=1;

$$\bar{W}(1) = 0 \tag{5.16}$$

$$\bar{W}'(1) = 0 \tag{5.17}$$

The $\bar{W}(\bar{X})$ can be approximated by the j^{th}–level wavelet series is given by:

$$\bar{W}(\bar{X}) = \sum_{k=2-N}^{2^j-1} \beta_k 2^{j/2}\, \varphi\left(2^j\bar{X} - k\right) \tag{5.18}$$

The integer j is used to control the smoothness of the solution. The integer j value is larger more accurate result will be obtained.The equation (5.17) is a function may be represented in terms of the Daubechies scaling function at scale J, β_k is the expansion coefficients give the magnitude of each scaling function component of the solution. Substituting the equation (5.17) in to (5.10).obtain wavelet galerkin discretization as follows:

$$2^{2J}\int_0^1 2^{J/2}\varphi''(2^J\bar{x}-k)2^{J/2}\varphi(2^J\bar{x}-m)d\bar{x}.\beta_k$$

$$+\bar{P}\int_0^1 2^{J/2}\varphi(2^J\bar{x}-k)2^{J/2}\varphi(2^J\bar{x}-m)d\bar{x}\,\beta_k$$

(5.18)

Where,

m= 2-N, 3-N,………,2^J-1

where.

$$\int_0^1 2^{J/2}\varphi''(2^J\bar{x}-k)2^{J/2}\varphi(2^J\bar{x}-m)d\bar{x}.$$

$$=\int_0^1 \varphi''(2^J\bar{x}-k)\varphi(2^J\bar{x}-m)d(2^J\bar{x}-m).$$

$$=\int_{-m}^{2^J-m} \varphi''(y-k+m)\varphi(y)dy$$

$$=\Delta_{k-m}^2(2^J-m)-\Delta_{k-m}^2(-m) \tag{5.19}$$

Similarly

$$\int_0^1 2^{J/2}\varphi(2^J\bar{x}-k)2^{J/2}\varphi(2^J\bar{x}-m)d\bar{x}$$

$$= \int_{0}^{1} \varphi(2^J\bar{x} - k)\varphi(2^J\bar{x} - m)d(2^J\bar{x} - m).$$

$$= \int_{-m}^{2^J-m} \varphi(y - k + m)\varphi(y)dy$$

$$= \Delta_{k-m}^{0}(2^J - m) - \Delta_{k-m}^{0}(-m) \quad (5.20)$$

Substituting equation (5.19) and (5.20) in equation (5.18), we have,

$$2^{2J}[\Delta_{k-m}^{2}(2^J - m) - \Delta_{k-m}^{2}(-m)]\beta_k + \bar{P}[\Delta_{k-m}^{0}(2^J - m) - \Delta_{k-m}^{0}(-m)]\beta_k = 0 \quad (5.21)$$

The values for calculating the connection coefficients explain in the chapter 3.

From the equation (5.14) to (5.17), W(x) satisfy the boundary condition as follows,

For x = 0;

$$\bar{W}(0) = \sum_{k=2-N}^{5-N} \beta_k \, \varphi(-k) = 0 \quad (5.22)$$

$$\bar{W}'(0) = \sum_{k=2-N}^{5-N} \beta_k \, \varphi'(-k) = 0 \quad (5.23)$$

For x = 1

$$\bar{W}(1) = \sum_{k=2^J-N+2}^{2^J-1} \beta_k \, \varphi(2^J - k) = 0 \quad (5.24)$$

$$\bar{W}'(1) = \sum_{k=2^J-N+2}^{2^J-1} \beta_k \, \varphi'(2^J - k) = 0 \quad (5.25)$$

The buckling load corresponding to the least eigenvalue problem. The equation (5.21) together with the equation (5.22) to (5.25) to form a over determined equation of β_k.

5.5.2. Simply Supported

The boundary condition for non dimensional clamped support is as follows

For x=0;

$$\bar{W}(0) = 0 \tag{5.26}$$

$$\bar{W}''(0) = 0 \tag{5.27}$$

For x=1;

$$\bar{W}(1) = 0 \tag{5.28}$$

$$\bar{W}''(1) = 0 \tag{5.29}$$

the derivation are similar to clamped support.W(x) satisfy the boundary condition as follows,

For x = 0;

$$\bar{W}(0) = \sum_{k=2-N}^{5-N} \beta_k \, \varphi(-k) = 0 \tag{5.30}$$

$$\bar{W}''(0) = \sum_{k=2-N}^{5-N} \beta_k \, \varphi'(-k) = 0 \tag{5.31}$$

For x = 1

$$\bar{W}(1) = \sum_{k=2^J-N+2}^{2^J-1} \beta_k \, \varphi(2^J - k) = 0 \tag{5.32}$$

$$\bar{W}''(1) = \sum_{k=2^J-N+2}^{2^J-1} \beta_k \, \varphi'(2^J - k) = 0 \tag{5.33}$$

The equation (5.18) can be writtern in another form as follows:

$$\sum_{k=2-N}^{2^J-1} C_{k,m}^{j} \, \beta_k + \sum_{k=2-N}^{2^J-1} D_{k,m}^{j} \, \beta_k = 0 \quad m = 2-N, 3-N \dots\dots\dots, 2^J - 1 \tag{5.34}$$

Where,

$$C_{k,m}^{j} = \int_0^1 2^{J/2} \varphi''(2^J \bar{x} - k) 2^{J/2} \varphi(2^J \bar{x} - m) d\bar{x}. \beta_k \tag{5.35}$$

$$D_{k,m}^{j} = \int_0^1 2^{J/2} \varphi \, (2^J \bar{x} - k) 2^{J/2} \varphi(2^J \bar{x} - m) d\bar{x}. \beta_k \tag{5.36}$$

The equation (5.34) can be expressed in matrix vector form as follows:

$$W\beta = 0 \tag{5.37}$$

$$W = 2^{2j} C + \bar{P} \, D \tag{5.38}$$

Where,

$$C = \left[C_{k,m}^{j}\right]_{2-N \le k,m \le 2^J-1} \tag{5.39}$$

$$D = \left[D_{k,m}^{j}\right]_{2-N \le k,m \le 2^J-1} \tag{5.40}$$

And $\beta = [\beta_{2-N} \beta_{3-N} \; \dots\dots.. \;\; \beta_{2^J-1}]^T$ (5.41)

Now we have a linear system of $2^J + N - 2$ equation of the $2^J + N - 2$ unknown coefficients. We can obtained the coefficient of the approximate solution by solving this linear system of equation. The solution of β gives the expansion coefficients of Wavelet Galerkin approximation.

5.6. RESULT AND DISCUSSION

The buckling load P_{cr} of structure is corresponding to the least eigenvalue of the problem. The equation (5.21) together with the boundary condition to form a over determined equation of β_k. In practical computation, QR decomposition is a powerful tool for solving least square fitting problem, to change the discretized equation to be an upper triangular matrix, let the determinate of matrix is zero, the least root of the eigenvalue problem is required as P_{cr}. The results are listed in table (5.1). Figure (5.2) shows the percentage of error for different levels of resolution.

Table (5.1): values of P_{cr} for N=6, J=0,1,3

Boundary condition	Wavelet galerkin method			FEM	Exact method
	J=0	J=1	J=3		
Simply supported	$\frac{9.58EI}{L^2}$	$\frac{9.65EI}{L^2}$	$\frac{9.685EI}{L^2}$	$\frac{9.8EI}{L^2}$	$\frac{9.8696EI}{L^2}$
Clamped supported	$\frac{37.21EI}{L^2}$	$\frac{37.78EI}{L^2}$	$\frac{38.21EI}{L^2}$	$\frac{40EI}{L^2}$	$\frac{39.478EI}{L^2}$

Table (5.2): Daubechies filter coefficients for N=6

k	a_k
0	0.0498
1	0.1208
2	-0.1909
3	-0.6504
4	1.1411
5	-0.4705

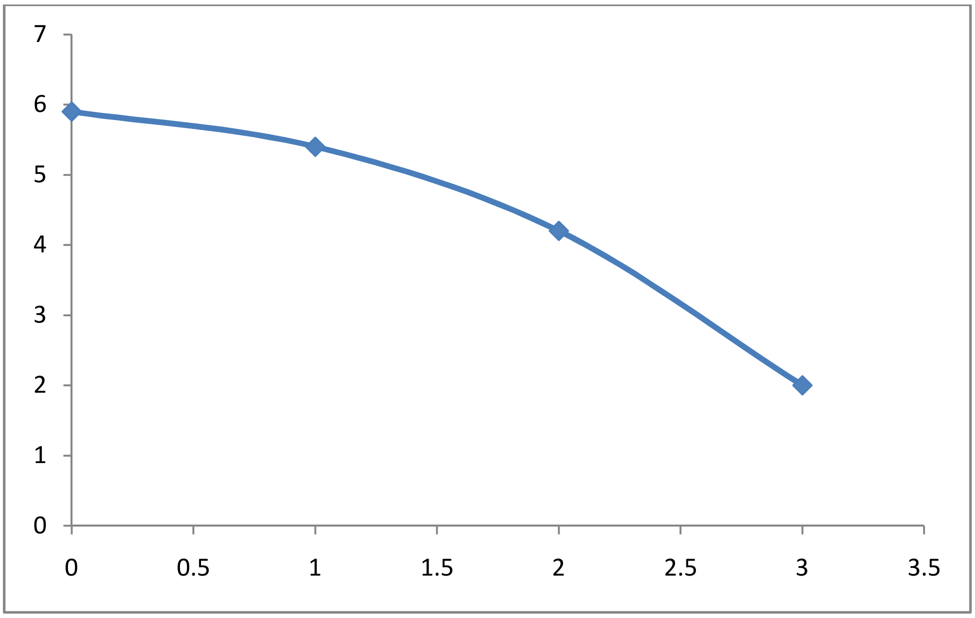


Figure (5.2): Percentage of error for different levels of resolution.

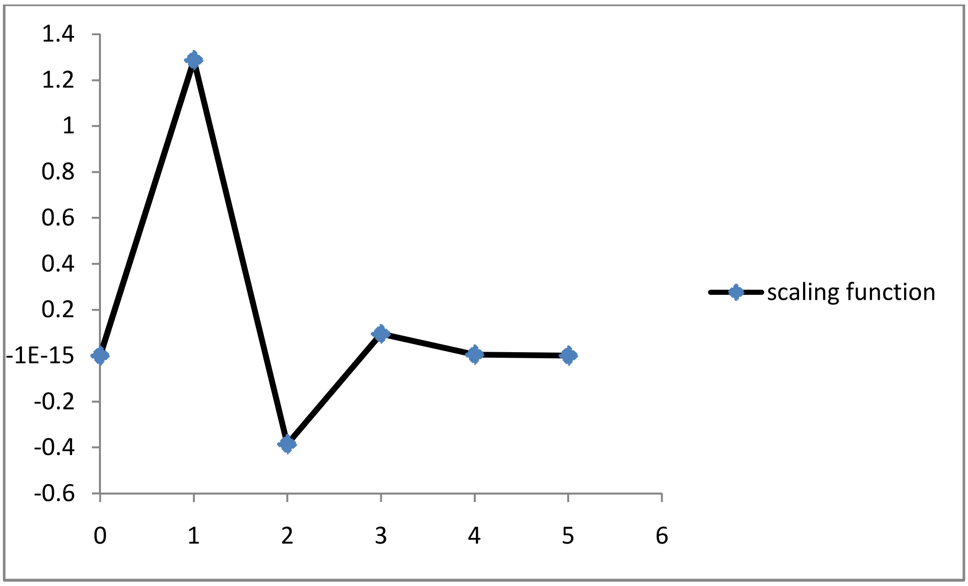


Figure (5.3): Scaling Function for N=6

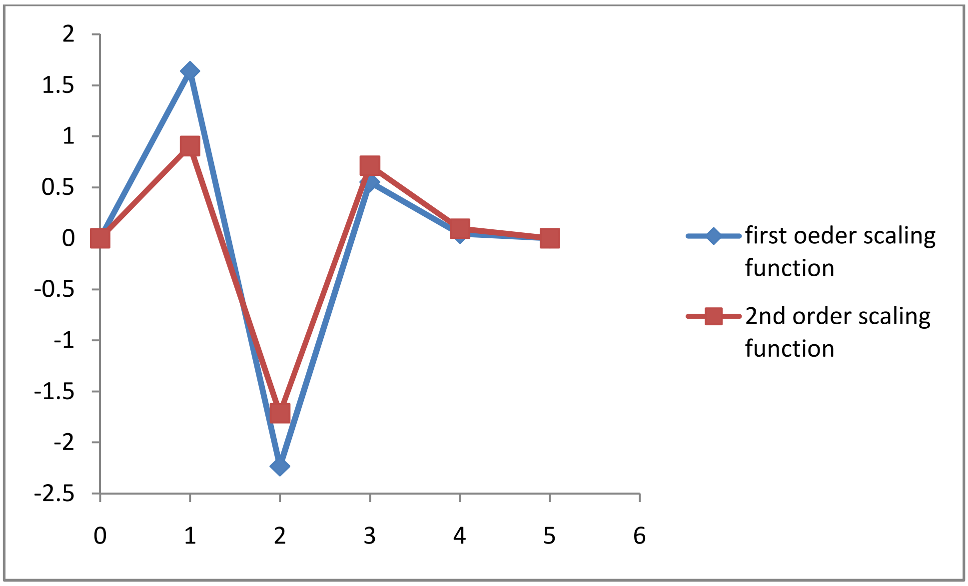


Figure (5.4): Derivatives of Scaling Function for N=6

CHAPTER 6

CONCLUSION

6.1. CONCLUSION

The Wavelet Galerkin method is a good alternative for solving partial differential equations. In present project work, the Daubechies family of wavelet have been consider because they posses several useful properties, such as orthogonality, compact support and ability to represent function at different levels of resolution. Wavelet galerkin Method has been shown to be a powerful numerical tool for accurate solution of partial differential equation. Wavelet Galerkin methods have more advantages than standard Galerkin methods by using compactly supported orthogonal functional basis. In wavelet Galerkin method, it uses Daubechies coefficients and the scaling functions. Daubechies wavelets are more useful in the numerical solution of ordinary differential equation and partial differential equation.

The wavelet Galerkin approximation relies on evaluation of connection coefficients. In present project work, described the algorithms for the exact evaluations of connection coefficients of Daubechies compactly supported wavelets. The buckling load of pressurized bar obtained using Daubechies 6 or D6 coefficients wavelets have been compared with the exact solution. This leads to considerable savings of the time and improves numerical results through the reduction of round of errors.

6.2. SCOPE OF FUTURE WORK

Through in this project work, some studies attempted for buckling of bar but still many areas are left which require further investigation. The possible extensions to the present study as below:

- In present project work, described the algorithms for the exact evaluations of connection coefficients of Daubechies compactly supported wavelets, These algorithm also used for finite domain problem.
- The present study deals with buckling of bar. This nay extended for bending of beams and plates.
- In the place of scaling functions, wavelet functions can also be used for this type of problems.

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